Manipulation and Information Acquisition

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1 Introduction

Information is essential for investors trading in financial markets. Better informed investors are able to take better decisions and obtain higher profits when trading with less informed investors. The production of information doesn't come cheap. It usually requires considerable costs in money and time and, above all, expertise that only a few possess. Yet, everyday we see valuable information, such as portfolio recommendations and price targets, being given away to the public through the media. The common explanation for this is that, once information has been used to start a trading strategy and/or has been sold to other investors, keeping the information private provides no additional benefit to its owner, whereas releasing it to the public enables him to cash-in quickly after prices fully reflect that information.

Although there is a honest motivation to give away information, there is also a strong incentive to manipulate it. Legal prosecution prevents those blatant cases of manipulation common before the Great Depression. But it is not very effective against less ambitious attempts of manipulation. Valuation of equity is not an exact science and, to some extent, can be tweaked to produce the desired valuation. And, of course, there is always the possibility of honest mistakes. Thus, most of the times it is up to investors who receive the information to judge its merits and, by doing so, discipline those who release information. Investors can do so essentially in two ways: analyzing the track record, or reputation, of the information issuer; and comparing the information released with other contemporaneous sources of information.

Several papers have analyzed information-based manipulation in settings where manipulators are kept in line due to reputation concerns (e.g. Benabou and Laroque 1992, van Bommel 2003 and Fishman 2007). However, to the best of my knowledge, none considers the existence of other sources of information that investors can use to assess the credibility of the information announced. In this paper I develop a model that attempts to fill this void.

The model is based on the Grossman and Stiglitz's (1980) model. I add an agent, the manipulator, who has imprecise private information on the liquidation value of the risky asset. He uses his information to open a position in the risky asset, and then makes an announcement based on his information. He may choose to announce truthfully or to manipulate his announcement. After the price incorporates the information contained in the announcement, he closes his position. Fully rational investors have the opportunity to purchase imprecise information, independent of the manipulator's information, at a cost. They can use their own information to infer the probability with which the announcement was manipulated, and extract information from the announcement accordingly.

Both the manipulator and investors face trade-offs. The manipulator wants to maximize his profit by strategically choosing what to announce. This means that he wants to maximize the price impact of his announcement which, sometimes requires him to manipulate the announcement. However, investors can use their own information to assess the credibility of the announcement. The less credible the announcement, the less weight investors put on it and so, the smaller the price reaction to the announcement. In turn, investors wish to substitute the costly information by the costless information provided by the announcement. However, by doing so investors decrease their ability to assess the credibility of the announcement. This results in more manipulation and in the deterioration of the information content of the announcement. In face of these trade-offs, it is not obvious what the equilibrium outcome should be.

In this paper I'm particularly interested in analyzing the manipulator's announcement strategy and how it impacts: (i) the investors' decision to purchase information; (ii) the price efficiency; and (iii) the risk premium. In essence, is the presence of the manipulator welcomed or not? If investors are rational and understand that the announcement may be manipulated, is manipulation bad enough so that it should be eradicated at all costs?

Another set of questions is related to the recent regulation changes, introduced with the objective of preventing manipulation of analyst reports. The NASD rule 2711 and NYSE rule 472 of 2002 introduce several changes with the objective to increase transparency of analyst reports and decrease the incentive to their manipulation. By clarifying what is allowed and what is not, it increases the scrutiny on those reports. In turn, the 2003 Global Settlement between regulators and several leading investment banks requires each bank to fund research by government approved independent analysts, and to append it to their own reports. This ruling effectively improves the ability of investors to judge the credibility of analyst reports released by investment banks. A natural question to ask is then how effective are these measures? Do they produce the intended results? Are there any undesirable side effects? Will the manipulator be pushed away from more regulated markets to less regulated markets?

To avoid spoiling the party I will not go through the usual laundry list of results here. Instead I will only give a peek at the results. First, being under the scrutiny of rational and informed investors, the manipulator would prefer to announce truthfully. The problem is that he cannot commit to do so. Thus, he welcomes almost anything that helps him commit to announcing truthfully. This includes penalties for manipulating and better informed investors, but there are exceptions. Second, I show that manipulation is not bad *per se* in terms of price efficiency and risk premium. It's just not as good as truthful announcements. Third, the regulation changes of 2002/03 are, in most circumstances, effective in preventing manipulation. But in some cases manipulation is prevented at the cost of making the manipulator not announcing, which has an undesirable impact on price efficiency and risk premium. Finally, in certain circumstances announcements may have a toll on price efficiency and risk premium when not all investors are informed. The reason is that announcements deteriorate the information that uninformed investors (those who do not purchase information) extract from prices. As strange as it might seem, for uninformed investors more information sometimes results in less precise beliefs.

I also use the model to derive testable implications of manipulation, and use them to assess the validity of the research design and conclusions drawn by Cliff (2007). Cliff uses the regulation changes of 2002/03 as a natural experiment to test for manipulation of reports by analysts affiliated to investment banks. He finds that, prior to the changes in regulation the price response to sell recommendation is larger than for buy or hold recommendations.

After the regulation changes, the price response to buy and hold recommendations improves and becomes closer to that of sell recommendations. Cliff takes this as evidence of upward manipulation which is mitigated by the regulation changes. The model predicts exactly the same. He also looks at the post-announcement performance of portfolios formed based on the recommendation, finding a negative performance in all portfolios. Again he takes it as evidence of upward manipulation. Although he reaches the correct conclusion, his interpretation is not correct. It is not the underperformance of the buy portfolio that is evidence of upward manipulation. It is the overperformance of the buy portfolio relative to the sell portfolio that suggests the existence of upward manipulation.

This paper adds to the literature in information-based manipulation. The closest references are the paper by Benabou and Laroque (1992), van Bommel (2003) and Fishman (2007). Benabou and Laroque (1992) and Fishman (2007) analyze the strategic disclosure of information by insiders in models of repeated interaction where investors learn about the type of the insiders. In Benabou and Laroque (1992) the insider always have inaccurate information and may be honest or opportunistic, in which case he occasionally manipulates the announcement. In Fishman (2007) the insider may be a charlatan, who does not have information, or a genuine leader, who has inaccurate information with positive probability. In both papers, insiders restrict the frequency of manipulation in order to build and maintain a reputation that allow them to influence ex-post prices. Similarly, in van Bommel (2003) an insider spreads rumors and, once again, with repeated interaction he refrains from spreading untruthful rumors for reputation concerns. In all these papers, investors do not have the possibility to obtain information from other sources in order to evaluate the truthfulness of the announcement. Therefore, insiders restrict their manipulation only because there is repeated interaction and reputation building. In one-shot games, insiders cannot credibly commit to be truthful and so the announcement is disregarded by investors. In the model developed in this paper, the existence of another source of information acts as a mechanism to deter the manipulator from being completely untruthful. As a result, partially informative announcements are credible in a one-shot game.

The paper proceeds as follows. Section 2 describes the model. Section 3 analyzes in depth the manipulator's announcement strategy and its consequences in a simplified setting where all investors observe additional information for free. Section 4 analyzes the investors' decisions to purchase information. Section 5 concludes. All proofs are provided in the Appendix.

2 Model Description

The model I develop in this section is a noisy rational expectations model (NREE) based on the Grossman and Stiglitz (1980) model with two modifications. First, I allow for the existence of a manipulator, whose announcements are an additional source of information. Second, I assume that the risky asset's liquidation value has a binomial distribution, instead of a normal distribution. This assumption will significantly simplify the analysis, because the manipulator can only be untruthful in one way.

In the remainder of this section I describe the model and discuss the assumptions made. The structure of the model and its parameters are common knowledge to all agents. Figure 1 provides a time line of events and summarizes the model.

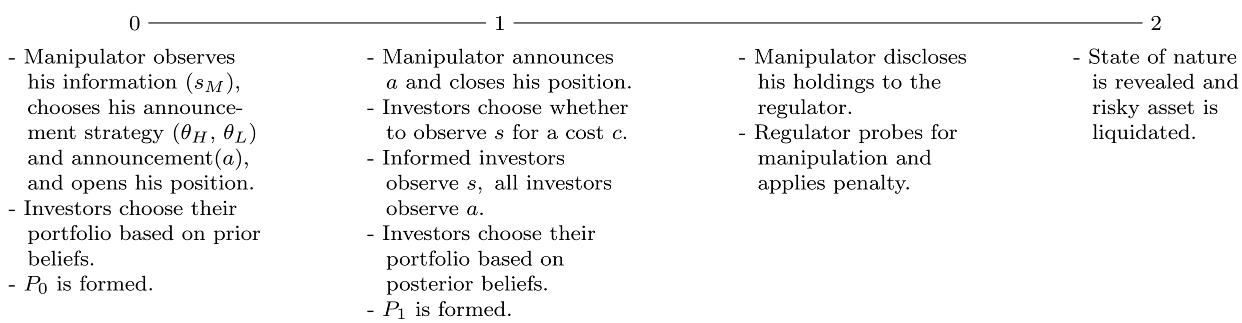

Figure 1: Timeline of events.

2.1 Investment Opportunities

There are two assets available for trading at dates 0, 1 and 2: one riskless asset with infinitely elastic supply and gross rate of return R, normalized to 1; one risky asset with liquidation at date 2. The liquidation value, V, is either V_H with probability q or V_L with probability $1 - q$. Without loss of generality I set $V_H = 1$ and $V_L = 0$.[1] For simplicity I will focus on a

[1]For any $V_H > V_L$ we can always write $V_H = a + b \times 1$ and $V_L = a + b \times 0$. Therefore the expected value and variance of V can be readily obtained from those obtained under the assumption that $V_H = 1$ and $V_L = 0$.

symmetric distribution, i.e. $q = 1/2$.

As is usual in the literature (see e.g. Grossman and Stiglitz, 1980), I assume that the risky asset's supply at dates 0 and 1, denoted by z_t, is random with distribution $z_t \sim \mathcal{N}\left(\bar{z}, \sigma_z^2\right)$, $\bar{z} \geq 0$. I further assume that z_1 is independent of z_0.

2.2 Information

The liquidation value of the risky asset is determined at date 0, but remains unobservable until date 2. However, there are two signals for the liquidation value available before date 2. Both signals tell whether state H or L will occur (i.e. whether the liquidation value will be V_H or V_L) with some accuracy.

The first signal is denoted by $s \in \{H, L\}$ and has accuracy $\rho \in [1/2, 1]$, meaning that the signal is correct with probability ρ.[2] It is produced immediately before date 1 trading occurs, and is available for all agents to observe at a cost c. Agents who observe the signal can use its information to trade at date 1. The second signal is denoted by $s_M \in \{H, L\}$ and has accuracy $\rho_M \in [1/2, 1]$. This signal is made available immediately before date 0 trading occurs and to one agent only, which I will call the manipulator. The manipulator can use the information of his signal to trade at date 0. Both signals are independent which implies that both $s = s_M$ and $s \neq s_M$ may occur.[3]

In addition, immediately before date 1 trading occurs, the manipulator may choose to make an announcement based on his signal, denoted by $a \in \{H, L, N\}$; $a = N$ means that the investor does not announce. The information announced can be different from the one that is observed by the manipulator, in which case there is manipulation. All agents observe the announcement and can use its information to trade at date 1. The observation of the announcement a and the decision to observe signal s is made simultaneously.[4] For this reason, the manipulator does not use the information of signal s when deciding what to announce.

[2] Because there are only two states of nature, an accuracy of 1/2 means that the signal has no information. Notice that a signal with accuracy below 1/2 is equivalent to a signal with accuracy equal to its complement and the opposite outcome.

[3] In extensions I will also consider the case where the most accurate signal cannot be incorrect if the least accurate signal is correct. In this case whenever the two signals disagree, the state of nature is revealed and corresponds to the signal of the most accurate signal.

[4] I will also consider extensions where agents decide whether to observe signal s after observing the announcement, and where the manipulator chooses what to announce after knowing the agents' decisions about the observation of signal s.

The existence of two different signals available to distinct types of agents at different times can be justified as follows. The signal s_M is the result of research performed by an analyst with vast resources which allow him to complete his research quicker than any other agent. Furthermore this analyst uses his research primarily for trading purposes. We can think of an analyst affiliated to an investment bank. In turn, signal s is the result of research performed by an analyst with less resources, which implies that he takes longer to complete his research. The lack of financial resources and the delay in the production of research reduces his profits from trading on his information. Therefore this analyst specializes in selling information to the public. We can think of an independent analyst. Cliff (2007) provides evidence that initial coverage of recently listed firms is 10 times faster for affiliated analysts (median of 1.5 months) than for independent analysts (median of 15 months). This evidence suggests that analysts with more resources produce research significantly faster than those with less resources.

Alternatively, signal s can be the result of research made by investors on their own. In this context, c represents the opportunity cost of the time spent in research. All investors are assumed to have the same ability so that their research produces the same signal.

The triple (s, a, s_M) completely characterizes the information environment. To normalize notation, variables specific to a given informational state or expected values conditional on the informational state are subscripted by (s, a, s_M). If the variable is independent of, say, s_M, then it is indexed by $(s, a, -)$. The same applies for expectations conditional on s and a but not on s_M.

2.3 Investors

There is a continuum of risk averse investors in the interval $[0, 1]$. All investors have the same preferences over date 2 wealth, W_2. At date 1 each investor chooses his demand for the risky asset, X_1, in order to maximize the expected utility of W_2 given his budget constraint. For this investors solve the problem

$$\begin{aligned} \max_{X_1} E\left[U\left(W_2\right) \mid \mathcal{F}_1\right] &= E\left(W_2 \mid \mathcal{F}_1\right) - \frac{\alpha}{2} Var\left(W_2 \mid \mathcal{F}_1\right) \\ & s.t. \quad W_2 = W_1 + X_1\left(V - P\right), \end{aligned} \tag{1}$$

where $\alpha > 0$ is the risk aversion parameter and $\mathcal{F}_1$ denotes the information available to the investor at date 1. The choice of these preferences is motivated only by tractability. The same motivation makes the use of CARA preferences and normally distributed liquidation values popular in the literature.[5] I do not believe the results are sensitive to the choice of preferences, as long as investors are risk averse. However, the use of other preferences would substantially complicate the computation of the equilibrium.

Investors are identical in every aspect except for the information they possess at date 1. Some investors pay the cost to observe signal s and so are better informed than those who don't. I will call the former informed investors and denote them by I and the latter uninformed investors and denote them by U. The fraction of informed investors is denoted by λ. Uninformed investors are sophisticated enough to extract information from the equilibrium price, which partially reflects the information contained in signal s. Therefore their date 1 information set is either $\mathcal{F}_1^U = \{a, P_1\}$ or $\mathcal{F}_1^U = \{P_1\}$, depending on whether the manipulator makes the announcement or not. P_1 denotes the date 1 equilibrium price. Informed investors possess all the information available to investors, and so there is no additional information for them to extract from the equilibrium price.[6] Therefore their date 1 information set is either $\mathcal{F}_1^I = \{a, s\}$ or $\mathcal{F}_1^I = \{s\}$. It is straightforward to see that a sufficient statistic for the investors' beliefs is the conditional probability they assign to state H, which I denote by $p_1^I \equiv Prob\left(H \mid \mathcal{F}_1^I\right)$ and $p_1^U \equiv Prob\left(H \mid \mathcal{F}_1^U\right)$.

At date 0 all investors have the same information set $\mathcal{F}_0$ corresponding to the prior on the states of nature. I assume that investors at date 0 do not anticipate the availability of additional information. This assumption allows the date 0 portfolio decision to be independent of the expected date 1 decisions. For the results in the paper this assumption is innocuous since I will focus on the date 1 equilibrium and treat the date 0 equilibrium price P_0 as a parameter.

[5] See for instance Grossman and Stiglitz (1980), Admati and Pfleiderer (1988) and Easley and O'Hara (2004). In this model the assumption of a binomially distributed liquidation value makes the expressions for the expected value of wealth and the risky asset's demand function for any commonly used utility function intractable at best.

[6] Informed investors do not observe the manipulator's signal s_M. However, the assumptions that I will make on the manipulator imply that his trading has no impact on the equilibrium price. Therefore investors cannot infer the manipulator's signal from the equilibrium price.

2.4 Manipulator

As mentioned previously, there is an agent who privately observes the signal s_M at date 0, which I call the manipulator. The manipulator is further characterized as follows: (i) he is risk neutral; (ii) he has measure zero; (iii) he has limited wealth and borrowing constraints;[7] (iv) the maximum size of his short positions is restricted to at most δ times the maximum size of his long positions; and (v) he faces liquidity constraints that force him to close his position in the risky asset at date 1.

Characteristics (ii) and (iii) imply that the manipulator is able to trade without being noticed. A similar assumption is used by van Bommel (2003). Kyle (1985) shows that informed agents ration their trade in order to minimize the price impact of their trade. Therefore, if the asset's trading volume is high, it is reasonable to assume that even some institutional investors can trade without impacting on price. Nonetheless, this assumption is inessential for the results of this paper and is made mainly for tractability. The only requirement is that investors cannot fully learn the manipulator's signal s_M from the price impact of his trade.

Regarding the short sales constraint, it is not clear cut what δ should be when there are no short sales constraints because of the margins. I will consider that there are no short sales constraints when $\delta = 1$. This corresponds to the case where the margin equals the value of the asset.[8] Without loss of generality I normalize the manipulator's wealth for the maximum size of a long (short) position to be 1 (δ).

In turn, the assumption of liquidity constraints provides a genuine motivation for the manipulator's announcement. The manipulator wants the date 1 price (at which he is forced to close his position) to be as far apart from the date 0 price (at which he opens his position) in the direction of his trade as possible. To that end, the manipulator can use his announcements to influence P_0, P_1, or both. However, for a reason that I will address in the next subsection, I restrict the manipulator to pre-announcement speculation (trading in anticipation of the

[7] It may seem counter intuitive that the manipulator has borrowing constraints but investors do not. One can always assume that investors face borrowing constraints but that their risk aversion is high enough and their borrowing constraints loose enough (even if tighter than those of the manipulator) such that their borrowing constraints will never bind and so can be ignored.

[8] If the margin is less than the value of the asset, then no short sales constraints would imply $\delta > 1$. However in such a case there is a possibility of bankruptcy which we can assume is very costly in order to justify the margin equal to the value of the asset.

announcement's effect) and rule out post-announcement speculation (induce erroneous beliefs and then trade based on private information).[9] Therefore, if the manipulator announces, he does so only at date 1 and only influences P_1. In this scenario, the manipulator is able to increase his profits by truthfully announcing his information, i.e. $a = s_M$, because his private information is incorporated into date 1 price. Hence, the manipulator may have a genuine motivation to announce truthfully. However, the optimal announcement strategy is not necessarily the truthful one. For instance, consider that the manipulator observes $s_M = L$. Short sale constraints or a low P_0 will significantly reduce the profit from the truthful strategy (short at date 0, announce $a = L$ at date 1) and will tempt the manipulator to do the opposite: take a long position at date 0 and announce $a = H$ in order to manipulate the date 1 price upward.

I will use $\theta_{a|s_M} \equiv Prob\,(a \mid s_M) \in [0,1]$, $a \in \{H, L, N\}$ and $s_M \in \{H, L\}$, to denote the probability with which the manipulator announces a conditional on the observation of signal s_M. Because the manipulator is risk neutral, he always take positions with a size equal to the maximum allowed. Therefore, $\mathcal{T}_{(-,a,s_M)} \in \{1, -\delta\}$ characterizes his trading strategy when he announces a and observes signal s_M.

The same assumption of forced liquidation is made by Fishman (2007) in a paper related to this one. More generally, the assumption of early liquidation risk appears in a large literature, e.g. DeLong et al. (1990) and Dow and Gorton (1994). There are several explanations for the early liquidation in the literature. Here I assume that early liquidation arises as a consequence of large opportunity costs (similar to Shleiver and Vishny, 1990) resulting from: (i) long time between dates 1 and 2; and (ii) better investment opportunities at date 1.

Finally, notice that signal s is useless to the manipulator since his announcement and s are released simultaneously and he exits the market immediately after that. Therefore, the manipulator ignores s.

[9] See Benabou and Laroque (1992) and van Bommel (2003) for models where both types of speculation are allowed.

2.5 Regulator

There is an agent, which I call the regulator, whose function is to identify and punish information-based manipulation. After date 1, the regulator investigates the existence of manipulation. At that time the regulator costlessly observes the signal s, equilibrium prices P_0 and P_1, and the manipulator's portfolio holdings at date 0 and 1, which the manipulator is required to disclose after date 1. The penalty for manipulation is $K > 0$ per unit traded. K is assumed to be sufficiently large to deter manipulation if it is punished for sure.

The regulator is aware that the manipulator faces liquidity constraints that force him to liquidate his position at date 1. Therefore, the regulator does not consider a long (short) position at date 0 followed by the announcement $a = H$ ($a = L$) and the closing of this position at date 1 as a proof of manipulation. Without further information, this behavior is consistent with the manipulator being truthful. By the contrary, if the regulator observes a long (short) position and announcement of $a = L$ ($a = H$) at date 0, and the closing of this position at date 1, the regulator knows there is manipulation. Clearly, in this case the announcement was made with the objective of allowing the manipulator to open his position at a more favorable price. The sure penalty of K that results from post-announcement speculation rules out this strategy and leaves pre-announcement speculation as (possibly) the only viable mean of manipulation. However, this does not mean that pre-announcement speculation will always escape unpunished. The punishment for this type of speculation depends on the type of the regulator.

I will consider that the regulator is of one of two types: skilled or unskilled. The skilled regulator can uncover the private information of the manipulator (s_M) with a small probability. He punishes the manipulator only if he uncovers s_M and finds out that $s_M \neq a$. The expected penalty for manipulation when the regulator is skilled is denoted by k (K times the probability the regulator uncovers s_M). On the other hand, the unskilled regulator is unable to uncover s_M, and thus doesn't have hard evidence against the manipulator. However, because he is under the pressure of public opinion to punish manipulation, he occasionally punishes the manipulator based on the discrepancy of the manipulator's announcement a and the liquidation value V.[10] I will also use k to denote the expected penalty with an unskilled regulator (K

[10]We can think that when the unskilled regulator is under pressure to deliver results he may threaten the

times the probability the regulator punishes).

Notice that whereas a skilled regulator forces the manipulator to manipulate less frequently, an unskilled regulator may force the manipulator to go silent. In the latter case, even though the manipulator announces truthfully he may still be punished if he is unlucky and $s_M \neq V$.

In both cases, whenever $k = 0$ it means that the regulator punishes pre-announcement manipulation with zero probability, and not that $K = 0$. That is, post-announcement manipulation is still punished and therefore ruled out.

2.6 Equilibrium Definition

The definition of the date 0 equilibrium is trivial, since there is not much going on at date 0. The equilibrium demand for the risky asset maximizes the expected utility of investors given their (prior) beliefs and the market clearing equilibrium price function $\boldsymbol{P}_0$ (the actual equilibrium price P_0 depends on the realization of the random supply, z_0).

The equilibrium price P_0 turns out to be the only date 0 equilibrium variable that influences the date 1 equilibrium. Long (short) positions are more profitable for the manipulator the smaller (larger) P_0 is, and so P_0 exerts an important influence on the manipulator's optimal strategy. Because I want to focus on the date 1 equilibrium, where all the action is concentrated, I will consider P_0 as a parameter in most of the paper. Notice that P_0 can take any value due to the normally distributed random supply z_0. To simplify notation, from now on I drop the time subscripts on all date 1 variables.

Since the equilibrium demands and price are functions of the random variables s, a, P and z, we obtain equilibrium demand and price functions, which I denote by $\boldsymbol{X}^I(s, a, P)$, $\boldsymbol{X}^U(a, P)$ and $\boldsymbol{P}(s, a, z)$. The investors' date 1 strategies are then fully described by $\mathcal{I} = \left(\boldsymbol{X}^I, \boldsymbol{X}^U, \lambda\right)$; λ aggregates the individual choices of each investor about whether to observe s or not. In turn, the manipulator's date 1 strategy is fully described by $\mathcal{M} = \left(\theta_{a|s_M}, \mathcal{T}_{(-,a,s_M)}\right)_{a,s_M}$. The definition of the date 1 noisy rational expectations equilibrium ($NREE$) is the following.

Definition 1 *A NREE with manipulator is a triple* $(\mathcal{I}^*, \mathcal{M}^*, \boldsymbol{P}^*)$ *such that:*

manipulator who, knowing that he has manipulated (now or in the past), agrees on a settlement and pays a penalty smaller than K in order to avoid further investigations that will damage his reputation.

(i) $\boldsymbol{X}^{I*}$ *(*$\boldsymbol{X}^{U*}$*) maximize the expected utility of informed (uninformed) investors given their posterior beliefs* p^I *(*p^U*) and* $\boldsymbol{P}^*$*;*

(ii) Informed (Uninformed) investors form their posterior beliefs p^I *(*p^U*) from the observation of signal* s *(price function* $\boldsymbol{P}^*$*) and announcement* a*, while taking into account the manipulator's optimal announcement strategy* $\theta^*_{a|s_M}$*; posterior beliefs in zero-probability events are pinned down by considering that the manipulator trembles are independent of* s_M*;*

(iii) λ^* *is such that no investor can improve his expected utility by changing his decision on whether to observe* s *or not;*

(iv) $\boldsymbol{P}^*$ *clears the market for the risky asset given* $\boldsymbol{X}^{I*}$*,* $\boldsymbol{X}^{U*}$ *and* λ^**;*

(v) $\theta^*_{a|s_M}$ *maximize the manipulator's expected utility given* $\boldsymbol{P}^*$*,* P_0*,* k *and the type of regulator.*

As we will see in the next sections, there are multiple equilibria. Some of them are equivalent, in the sense that the payoff to all agents is the same. I call the collection of all equivalent equilibria a type of equilibrium. Because the manipulator is monopolistic and investors are infinitely numbered, I consider that all agent coordinate in the type of equilibrium that maximizes the manipulator's expected utility, which is the focal type of equilibrium.

3 Equilibrium without Information Acquisition

In this section I analyze the date 1 equilibrium when all investors observe the signal s for free. This case is akin to independent analysts making their recommendation available to the public for free. I defer the analysis of the general case, where investors decide whether to purchase s or not, until the next section. The reason to proceed in this way is twofold. First it allows me to analyze the manipulator's equilibrium behavior, which is essentially the same as in the general case, in a simpler setting. The next section will then be devoted mostly to the investors' equilibrium behavior, specifically their information acquisition decision.

Second, this setting is the closest to the one considered in Cliff (2007). Cliff conducts an empirical analysis of the stock price reaction, and subsequent performance, to public stock recommendations issued by affiliated analysts versus those issued by independent analysts. In his setting it is assumed that affiliated analysts (here the manipulator) have a conflict of interests which biases their recommendations, whereas independent analysts (here whoever sends signal s) do not have any conflict of interests and so their recommendations truthfully reflect their opinions.[11] The analysis of the equilibrium in this section can then be used to validate Cliff's research design and results, and suggest other testable implications of manipulation.

3.1 Solving for the Equilibrium

From the first order condition of the optimization problem 1 one obtains the usual linear demands for the risky asset

$$X^I = \frac{E\left(V \mid \mathcal{F}^I\right) - P}{\alpha Var\left(V \mid \mathcal{F}^I\right)} = \frac{p^I - P}{\alpha p^I\left(1 - p^I\right)}.$$

Market clearing then implies that

$$P\left(s, a, z\right) = p^I_{(s,a,-)} - p^I_{(s,a,-)}\left(1 - p^I_{(s,a,-)}\right)\alpha z,\ \forall s, a \tag{2}$$

where $p^I_{(s,a,-)}$ is short hand notation for the conditional investors' beliefs in the information scenario $(s, a, -)$. Beliefs $p^I_{(s,a,-)}$ for each of the 6 information scenarios are straightforward to obtain, and are presented in Appendix A.

In turn, the manipulator chooses his strategy in order to maximize his expected utility conditional on his private information, given by

$$\begin{aligned} E\left(U_M\right) &= \frac{E\left[U_M \mid s_M = H\right] + E\left[U_M \mid s_M = L\right]}{2} \\ E\left[U_M \mid s_M\right] &= \sum_{a \in \{H,L,N\}} \theta_{a|s_M}\Pi_{(-,a,s_M)},\ s_M \in \{H, L\} \end{aligned}$$

where $\Pi_{(-,a,s_M)}$ denotes the (normalized) profit from trading in state (a, s_M), which is given by

$$\Pi_{(-,a,s_M)} = \begin{cases} \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)}\left[\bar{P}_{(-,a,s_M)} - P_0\right] & if\, a = s_M \vee a = N \\ \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)}\left[\bar{P}_{(-,a,s_M)} - P_0 - k\right] & otherwise \end{cases}$$

[11] In this paper I consider only one manipulator. Therefore, there is at most one affiliated analyst for each stock. By the contrary, each stock may have more than one independent analyst issuing recommendations. Signal s is then the aggregation of all the recommendations issued by those independent analysts.

when the regulator is skilled and by

$$\Pi_{(-,a,s_M)} = \begin{cases} \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)} \left[\bar{P}_{(-,a,s_M)} - P_0\right] & if\, a = N \\ \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)} \left[\bar{P}_{(-,a,s_M)} - P_0 - k\left(1-\rho_M\right)\right] & if\, a = s_M \\ \max_{\mathcal{T}_{(-,a,s_M)} \in \{1,-\delta\}} \mathcal{T}_{(-,a,s_M)} \left[\bar{P}_{(-,a,s_M)} - P_0 - k\rho_M\right] & otherwise \end{cases}$$

when the regulator is unskilled. In the above, $\bar{P}_{(-,a,s_M)} \equiv E\left(P \mid a, s_M\right)$ is the expected price conditional on a and s_M.

Notice that when the regulator is unskilled, the manipulator may be punished even when he is truthful but, unluckily, is information is wrong (which occurs with probability $1-\rho_M$); however, he is not punished when he lies but is information his wrong (which occurs with probability ρ_M). Therefore, unlike in the case of a skilled regulator, being truthful does not guarantee that there is no punishment. The only way to avoid punishment when the regulator is unskilled is by not announcing.

Despite the fact that all these expressions have an explicit form, there is no closed form solution to the equilibrium. The equilibrium is computed numerically using McKelvey's (1992) algorithm.[12]

3.2 The Manipulator's Optimal Strategy

Before I characterize the manipulator's optimal strategy, I need to introduce some additional notation. $\bar{P}^T_{(-,a,s_M)}$ will denote $\bar{P}_{(-,a,s_M)}$ when the manipulator's announcement is believed to be truthful, and $\bar{P}^N_{(-,-,s_M)}$ will denote $\bar{P}_{(-,-,s_M)}$ when the manipulator never announces or, equivalently, his announcement is believed to be completely uninformative.

The next theorem provides a characterization of the manipulator's optimal announcement strategy when the regulator never punishes pre-announcement manipulation, i.e. $k = 0$. The associated optimal trading strategy is not relevant for the analysis and so is omitted. Interested readers can find it in the proof of the theorem.

Theorem 1 *If the regulator never punishes pre-announcement manipulation ($k = 0$), then*

[12] See Judd (1998) pp. 133-135 for a description of McKelvey's algorithm.

there exist $\bar{P}^1 \leq \bar{P}^2 \leq \bar{P}^3 \leq \bar{P}^4$ *defined by*

$$\bar{P}^1 = \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta}, \bar{P}^2 = \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta},$$
$$\bar{P}^3 = \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta}, \bar{P}^4 = \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)}}{1+\delta}$$

such that the optimal announcement strategy is

$$\begin{cases} \theta_{H|H} = \theta_{L|L} = 1, & if\ P_0 \in \left[\bar{P}^2, \bar{P}^3\right]\ (Truthful\ Announcement) \\ \theta_{H|H} = 1,\ \theta_{H|L} = \omega_1,\ \theta_{L|L} = 1 - \omega_1 & if\ P_0 \in \left(\bar{P}^1, \bar{P}^2\right)\ (Upward\ Manipulation) \\ \theta_{L|L} = 1,\ \theta_{L|H} = \omega_2,\ \theta_{H|H} = 1 - \omega_2 & if\ P_0 \in \left(\bar{P}^3, \bar{P}^4\right)\ (Downward\ Manipulation) \\ \theta_{N|H} = \theta_{N|L} = 1 & if\ P_0 \in \left(-\infty, \bar{P}^1\right] \cup \left[\bar{P}^4, +\infty\right)\ (Never\ Announce) \end{cases}$$

where $\omega_1, \omega_2 \in (0,1)$, $\lim_{P_0 \uparrow \bar{P}^2} w_1 = \lim_{P_0 \downarrow \bar{P}^3} w_2 = 0$ *and* $\lim_{P_0 \downarrow \bar{P}^1} w_1 = \lim_{P_0 \uparrow \bar{P}^4} w_2 = 1$.[13]

Proof. See Appendix B. ■

The main result of the theorem is that the manipulator may find it optimal to announce truthfully for some values of P_0 despite manipulation remaining unpunished. Except for a knife-edge case we will see below, this is possible only because investors have an additional source of information (signal s). When s does not exist or, equivalently, is completely uninformative (i.e., $\rho = 0.5$), the manipulator's announcement induces the same expected equilibrium price regardless of s_M. This is so because investors don't observe s_M or any signal correlated with it.[14] Consequently, whatever the manipulator finds optimal when $s_M = H$ is also optimal when $s_M = L$ and he deviates from the truthful announcement. The only exception is in the knife-edge case where he is indifferent between $a = H$ and $a = L$. On the other hand, when s exists and is informative, the expected price will be larger when $s = H$ than when $s = L$, all else equal. Because it is more likely that $s_M = s$ than otherwise, the same holds true for s_M in place of s. This introduces a bias towards $a = s_M$ which results in truthful announcement for a range of P_0 values.

Naturally, if P_0 is sufficiently low (high), the manipulator will find optimal to deviate from

[13] ω_1 *and* ω_2 *correspond to twice the frequency of manipulation.*
[14] s and s_M are correlated whenever $\rho > 1/2$ and $\rho_M > 1/2$.

the truthful announcement even if s is informative, and will manipulate the announcement upward (downward). For such P_0 values, if investors believe the manipulator's announcement strategy to be truthful, then the manipulator would like to always announce $a = H$ ($a = L$). However, if he does so, his announcement is uninformative and has no impact on expected equilibrium prices (equivalent to not announcing). Although this is always an equilibrium, the manipulator can do better if he plays the most informative announcement strategy to which he can commit to, since the profits from his long (short) position on the asset increases whenever he influences the equilibrium price upward (downward).

In the case of upward manipulation this equates to being truthful conditional on $s_M = H$ (always announce $a = H$ with long position) and occasionally manipulating conditional on $s_M = L$ (mix between $a = H$ with long position and $a = L$ with short position). Thus, when investors observe $a = L$ they know that $s_M = L$ (the manipulator is being truthful), and give more weight to the announcement than when they observe $a = H$ because there is the chance that the announcement was manipulated (when $s_M = L$). As a consequence, the expected price when $a = H$ (and hence the profit from a long position) decreases relatively to that in a truthful announcement strategy whereas the expected price when $a = L$ (and hence the profit from a short position) remains unchanged. Therefore, if P_0 is not too low, there is an informative upward manipulation equilibrium where the manipulator is indifferent between $a = H$ or $a = L$ when $s_M = L$. The smaller P_0, the less informative the announcement is. If P_0 is sufficiently low, the manipulator ends up making an uninformative announcement or, equivalently, no announcement.

Now, I look at what changes in the manipulator's optimal strategy when a skilled regulator punishes pre-announcement manipulation.[15]

Theorem 2 *If the regulator is skilled and punishes pre-announcement manipulation ($k > 0$),*

[15] The optimal announcement strategy when the regulator is unskilled is more involved to fully characterize. Since I only want to make the point that the way the manipulator is punished matters, when the manipulator is unskilled I only look at the equilibrium announcement strategies when k is low or k is very large. The results for the case of an unskilled regulator are presented in the proof of theorem 4.

then there exist $\bar{P}^1 \leq \bar{P}^2 \leq \bar{P}^{2.5} \leq \bar{P}^3 \leq \bar{P}^4$ *defined by*

$$\begin{aligned}
\bar{P}^1 &= \begin{cases} \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}^*_{(-,H,H)}}{1+\delta} & if\ \bar{P}^1 > \bar{P}^T_{(-,L,L)} \\ -\infty & otherwise \end{cases} \\
\bar{P}^2 &= \begin{cases} \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta} & if\ k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ -\infty & otherwise \end{cases} \\
\bar{P}^{2.5} &\in \left[\frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta}, \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta} \right] \\
\bar{P}^3 &= \begin{cases} \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)} + \delta k}{1+\delta} & if\ k < \bar{P}^N_{(-,-,H)} - \bar{P}^T_{(-,L,H)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \\ \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^T_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)}}{1+\delta} & otherwise \end{cases} \\
\bar{P}^4 &= \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^*_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)}}{1+\delta},
\end{aligned}$$

where $\bar{P}^*_{(-,H,H)}$ $(\bar{P}^*_{(-,L,L)})$ *is the equilibrium value in the Upward (Downward) Manipulation equilibrium when* $P_0 = \bar{P}^1$ $(P_0 = \bar{P}^4)$, *such that that the optimal announcement strategy is*

$$\begin{cases} \theta_{H|H} = 1,\ \theta_{L|L} = 1 - \varepsilon,\ \theta_{N|L} = \varepsilon & if\ P_0 \in \left[\bar{P}^2, \bar{P}^{2.5}\right]\ (Truthful\ Announcement) \\ \theta_{H|H} = 1 - \varepsilon,\ \theta_{N|H} = \varepsilon,\ \theta_{L|L} = 1, & if\ P_0 \in \left[\bar{P}^{2.5}, \bar{P}^3\right]\ (Truthful\ Announcement) \\ \theta_{H|H} = 1,\ \theta_{H|L} = \omega_1,\ \theta_{L|L} = 1 - \omega_1 - \varepsilon,\ \theta_{N|L} = \varepsilon & if\ P_0 \in \left(\bar{P}^1, \bar{P}^2\right)\ (Upward\ Manipulation) \\ \theta_{L|L} = 1,\ \theta_{L|H} = \omega_2,\ \theta_{H|H} = 1 - \omega_2 - \varepsilon,\ \theta_{N|H} = \varepsilon & if\ P_0 \in \left(\bar{P}^3, \bar{P}^4\right)\ (Downward\ Manipulation) \\ \theta_{N|H} = \theta_{N|L} = 1 & if\ P_0 \in \left(-\infty, \bar{P}^1\right] \cup \left[\bar{P}^4, +\infty\right)\ (Never\ Announce) \end{cases}$$

where $\omega_1, \omega_2 \in (0,1)$, $\lim_{P_0 \uparrow \bar{P}^2} w_1 = \lim_{P_0 \downarrow \bar{P}^3} w_2 = 0$ *and* $\varepsilon \gtrapprox 0$.

Proof. See Appendix B. ■

When the penalty k is relatively small, the optimal announcement strategies are similar to those we obtained when $k = 0$. In fact, as k converges to zero, the strategies converge. The difference is that the manipulator's announcement strategy for a given P_0 is weakly more informative the larger k is, as the theorem below formalizes. This is so because the penalty acts as a commitment device for the manipulator; he can commit to more informative announcement strategies because manipulation is punished and so deviation to a less informative announcement is costlier in the presence of the penalty. Just by looking at the expressions for $\bar{P}^2$ and $\bar{P}^3$ it is easy to see that the range of P_0 values for which the announcement strategy is truthful widens as k increases.

When k becomes very large, any manipulation becomes prohibitively costly. Then, not surprisingly, the manipulator either announces truthfully or does not announce. In both cases he avoids the penalty altogether. The interesting aspect is that the manipulator chooses to announce truthfully for smaller values of P_0 and to not announce for large values of P_0. This is due to investors risk aversion, the existence of a positive average amount of risk and the reduction in uncertainty associated to a truthful announcement. If P_0 is sufficiently small, the manipulator always takes a long position in the asset regardless of what he announces. As a result, he is better off by announcing truthfully in order to benefit from the price increase associated to the reduction in uncertainty. Obviously, when P_0 is very large, he always takes short positions, and so he is not interested in reducing uncertainty and, consequently, does not announce.

Although the existence of a penalty favors more informative announcements, which in principle is desirable, one concern is that a large expected penalty may drive the manipulator away to less regulated markets/assets, that is, with smaller k. Curiously, the next theorem states precisely the opposite. If given the choice between entering in two markets/assets, he would always choose the one that is more heavily regulated, provided that the regulator is skilled.

Theorem 3 *If the regulator is skilled, increases in k weakly increase the manipulator expected utility and weakly improve the informativeness of the manipulator's announcement $\forall P_0$. If k is small enough so that the subset of P_0 values that support either a UM or a DM equilibrium is nonempty, then increases in k strongly increase the manipulator expected utility and strongly improve the informativeness of the manipulator's announcement for at least the P_0 values in that subset.*

Proof. See Appendix B. ■

This surprising result is quite simple to understand. Because investors can rationally anticipate the manipulator's equilibrium announcement strategy, manipulation is never a good deal for the manipulator. The more he manipulates, the less credibility investors attach to the announcement, and thus the less he can influence equilibrium prices. The only reason he manipulates is because he cannot credibly commit to do otherwise. If the manipulator could commit to be truthful, he would be better off doing so. To understand this, recall the previous discussion of the upward manipulation announcement strategy. By occasionally manipulating when $s_M = L$, the manipulator lowers the expected utility when $a = H$ regardless of s_M, while keeping the expected utility when $a = L$ unchanged. The equilibrium is attained when he is indifferent between both announcements when $a = L$. Therefore, if $s_M = L$ the manipulator obtains the same expected utility he would obtain if he could commit to being truthful, but *less* expected utility when $s_M = H$. It is then quite obvious that the manipulator prefers to operate in heavily regulated markets, which help him commit to truthful announcements, provided he is punished only by his wrongdoing, and not by bad luck.[16] However, if the regulator is unskilled, things are not so straightforward.

Theorem 4 *If the regulator is unskilled, k is relatively small, and ρ_M is not too small, increases in k weakly increase the manipulator's expected utility and weakly improve the informativeness of the manipulator's announcement $\forall P_0$. However, if k becomes large enough, the manipulator never announces and his expected utility decreases.*

Proof. See Appendix B. ■

When the regulator is unskilled, the manipulator is punished in two occasions: (i) when he manipulates and his information is correct and (ii) when he *does not* manipulate but his information is incorrect. It is obvious that the only way to avoid being punished is to not announce. Hence, as k gets really large, the manipulator never announces. When k is small, however, increases in k improve the commitment device in the same way as in the case of a skilled regulator. Hence, the manipulator is better off and his announcement is more informative in a slightly regulated market . This is so because although the manipulator is (on average) punished for announcing truthfully, he is punished more heavily for manipulating.

[16] The expected utility when $s_M = L$ is independent of the penalty for manipulation, since in that case the manipulator either does not manipulate or is indifferent between manipulating or not. However, the larger the penalty, the larger the expected utility when $s_M = H$ (up to a certain point) because the manipulator never manipulates in that case and the announcement $a = H$ is more credible.

This shows that the easier/cheaper to implement penalties based on the comparison between what is announced and the realized outcome are only an imperfect substitute for penalties based on the *de facto* manipulation. Both are successful in preventing manipulation. But the cheaper alternative only improves the informativeness of the announcement up to a certain point. After that, manipulation is reduced but at the cost of no announcements.

Next, I analyze how $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$, which define the region of P_0 values where each type of equilibrium occurs, change with the investors' signal accuracy (ρ), manipulator's signal accuracy (ρ_M) and short selling constraints (δ). For the sake of completeness, I also include the effect of changes in the penalty (k), when the regulator is skilled (or when there is no penalty), which was discussed above.

Theorem 5 *If the regulator is skilled or the regulator does not punish pre-announcement manipulation ($k = 0$), then*

(i) $\frac{\partial \bar{P}^3}{\partial \rho} \geq 0$, $\frac{\partial \bar{P}^4}{\partial \rho} \geq 0$, $\frac{\partial\left(\bar{P}^3 - \bar{P}^2\right)}{\partial \rho} \geq 0$, $\frac{\partial\left(\bar{P}^4 - \bar{P}^1\right)}{\partial \rho} \geq 0$, $\bar{P}^3 = \bar{P}^2$ if $\rho = 1/2$, and $\bar{P}^1 = \bar{P}^2 \wedge \bar{P}^3 = \bar{P}^4$ if $\rho = 1$ and $\rho_M < 1$;

(ii) $\frac{\partial \bar{P}^1}{\partial \rho_M} \leq 0$, $\frac{\partial \bar{P}^4}{\partial \rho_M} \geq 0$, $\frac{\partial\left(\bar{P}^4 - \bar{P}^1\right)}{\partial \rho_M} \geq 0$, $\frac{\partial\left(\bar{P}^3 - \bar{P}^2\right)}{\partial \rho_M} \geq (\leq)\, 0$ if ρ_M is small (large), $\bar{P}^1 = \bar{P}^2 = \bar{P}^3 = \bar{P}^4$ if $\rho_M = 1/2$ and $\bar{P}^2 = \bar{P}^3$ if $\rho_M = 1 > \rho$;

(iii) $\frac{\partial \bar{P}^j}{\partial \delta} < 0$, $j = 1, ..., 4$, $\frac{\partial \bar{P}^2 - \bar{P}^1}{\partial \delta} < 0$, $\frac{\partial \bar{P}^3 - \bar{P}^2}{\partial \delta} < 0$, $\frac{\partial \bar{P}^4 - \bar{P}^3}{\partial \delta} > 0$ and $\frac{\partial \bar{P}^4 - \bar{P}^1}{\partial \delta} < 0$;

(iv) $\frac{\partial \bar{P}^1}{\partial k} \leq 0$, $\frac{\partial \bar{P}^2}{\partial k} \leq 0$, $\frac{\partial \bar{P}^3}{\partial k} \geq 0$, $\frac{\partial \bar{P}^4}{\partial k} \geq 0$, $\frac{\partial\left(\bar{P}^3 - \bar{P}^2\right)}{\partial k} \leq 0$, and $\frac{\partial\left(\bar{P}^4 - \bar{P}^1\right)}{\partial k} \leq 0$.

Proof. See Appendix B. ■

The first part of the theorem confirms the fact, discussed previously, that a truthful announcement strategy equilibrium is knife-edge when investors have no additional source of information ($\rho = 1/2$).

As I explained before, when ρ increases from $\rho = 1/2$, expected prices conditional on $s = H$ become larger than those conditional on $s = L$. Moreover, the larger ρ is, the larger the prices conditional on $s = H$ and the smaller the prices conditional on $s = L$ become. At the same time, since $s_M = s$ is more likely than $s_M \neq s$, it follows that expected prices conditional on $s_M = H$ also become larger than those conditional on $s_M = L$. Recall that these are the expected prices the manipulator takes in account when choosing his optimal announcement and trading strategy. Then, for a given P_0, the larger (smaller) the expected prices when

$s_M = H$ ($s_M = L$), the more likely it is that announcing $a = H$ ($a = L$) and taking a long (short) position is optimal. Consequently, when the difference between expected prices conditional on $s_M = H$ and $s_M = L$ increases, the range of P_0 values for which a truthful announcement is optimal increases. As stated in the theorem, this is exactly what happens when ρ increases. As one can easily figure out, this follows directly from the fact that prices conditional on $s = H$ ($s = L$) increase (decrease) when ρ increases.

However, the mechanism through which prices conditional on s affect prices conditional on s_M is not so straightforward. Since the manipulator decides his strategy before s is known, he has to condition on s_M which is only imperfectly correlated with s. Because both signals can be wrong (except when $\rho = \rho_M = 1$), there is always a chance that $s \neq s_M$, although $s = s_M$ is more likely; the likelihood of $s = s_M$ increases as either ρ or ρ_M increase. Then, the expected price conditional on s_M is a function of the expected prices conditional on $s = H$, which increases in ρ, and $s = L$, which decreases in ρ. Now it becomes apparent that when ρ changes, the expected price conditional on s_M and s do not necessarily change in the same direction. It can be shown that $\bar{P}^T_{(-,H,H)}$, $\bar{P}^T_{(-,L,H)}$ and $\bar{P}^T_{(-,L,L)}$ increase in ρ, whereas $\bar{P}^T_{(-,H,L)}$ may increase or decrease. Hence, $\bar{P}^3$ increases in ρ but the same can't say what happens to $\bar{P}^2$.

The implication is that, although we can say that the range of P_0 values for which the truthful announcement is optimal increases, we can't guarantee that if a given P_0 supports a truthful equilibrium it will continue to do so when ρ increases. That is, the region of truthful announcement increases with ρ but may shift toward larger values of P_0 (the case when $\bar{P}^2$ increases). This in turn implies that an increase in ρ does not guarantee that a truthful announcement is more likely (recall that P_0 is normally distributed by virtue of the time 0 random supply).

Looking at the expression for $\bar{P}^2$, we can see that it decreases in ρ if $\bar{P}^T_{(-,H,L)}$ decreases by more than $\delta\bar{P}^T_{(-,L,L)}$ increases. This can be achieved if ρ_M is large enough because then $\bar{P}^T_{(-,L,L)}$ increases less and $\bar{P}^T_{(-,H,L)}$ decreases more in ρ. This is so for two reasons. First, when ρ_M increases, the likelihood of $s = s_M$ increases. Thus, when $s_M = L$ the manipulator increases the weight on the prices conditional on $s = L$, which decrease in ρ. Second, when ρ_M is large, investors put a large weight on a relatively to s when forming their beliefs. This implies that when ρ increases, the change in beliefs is much larger when $a \neq s_M$ than when $a = s_M$.[17] As a result, $\bar{P}_{(L,H,-)}$ decreases considerably more than what $\bar{P}_{(H,H,-)}$ increases and so $\bar{P}^T_{(-,H,L)}$ decreases more with ρ. At the same time, $\bar{P}_{(H,L,-)}$ increases more than what $\bar{P}_{(L,L,-)}$ decreases, which implies that $\bar{P}^T_{(-,L,L)}$ increases more with ρ. But, since prices conditional on $s = L$ are weighted more heavily, the net effect is that a larger ρ_M contributes to $\bar{P}^2$ decreasing in ρ. Finally, note that if δ is small, the weight on $\bar{P}^T_{(-,L,L)}$ on $\bar{P}^2$ is relatively small. Therefore $\bar{P}^T_{(-,H,L)}$ doesn't need to decrease by much to compensate for the increase in $\bar{P}^T_{(-,L,L)}$, which makes it easier to find that $\bar{P}^2$ decreases in ρ.

The case of informative announcements is very similar to truthful announcements. The range of P_0 values for which an informative announcement is supported increases in ρ. However, informative announcements are not necessarily more likely as ρ increases. For that to happen, ρ_M has to be relatively large and/or δ small.

Notice that when $\rho = 1$ (and as long as $\rho_M < 1$), the manipulator's announcement strategy is irrelevant. Moreover, numerical computations show that as $\rho > \rho_M$ converges to 1 the manipulator manipulates less frequently. This occurs because when ρ is large investors can accurately identify manipulation and their signal is so accurate that there is little information to be extracted from the announcement. Therefore, the manipulator has a very limited ability to influence prices through manipulation when investors are very well informed. So, the manipulator mainly resorts to announcing truthfully or not announcing. In the limit, when $\rho = 1$, the manipulator's announcement becomes irrelevant for investors, and so he his indifferent between any announcement strategy.

The following corollary, left unproved, summarizes the results discussed above.

Corollary 1 *If the manipulator is relatively well informed (i.e. ρ_M is large) and/or he faces considerable short sales constraints (δ is small), then both truthful announcements and informative announcements are more likely as ρ increases. Moreover, when $\rho > \rho_M$, there is a ρ value above which further increases in ρ reduce the frequency of manipulation, until the point where there is no manipulation, when $\rho = 1$. If ρ_M is large (small), the reduction in*

[17] If ρ_M is small, investors put a large weight on their signal, and so their beliefs are roughly the same regardless of what is announced. Therefore, an increase in ρ does not change the investors' beliefs by much and, more importantly, the change in beliefs is not much larger when $a \neq s_M$ than when $a = s_M$.

the frequency of manipulation is accompanied by an increase in the frequency with which the manipulator announces truthfully (does not announce).

As we have seen before, the manipulator prefers to operate in highly regulated markets because the penalty helps him commit to announce truthfully. Since an increase in ρ (when ρ_M is large) can be used as a substitute for an increase in k (when the regulator is skilled) to induce more informative announcements, a natural question to ask is "Does the manipulator prefer to enter in a market with better informed investors?". The answer is not so obvious as it may seem at first sight. As ρ increases the manipulator announces more truthfully but at the same time his announcements provide investors with less additional information. The next theorem answers the question affirmatively.

Theorem 6 *Whenever both truthful and informative announcements are more likely as ρ increases, the manipulator's expected utility increases in ρ unless P_0 is so large that the manipulator always takes a short position. Therefore, the manipulator is, on average, better off the larger ρ is.*

Proof. See Appendix B. ■

Turning the attention to what happens when ρ_M increases, the second part of the theorem 5 says that the probability of making an informative announcement increases in ρ_M. For any informative announcement strategy, the larger ρ_M, the more weight investors put on the announcement. Hence, the larger the manipulator's ability to influence prices. Naturally, this makes the manipulator more prone to take advantage of his ability by announcing. Looking at the expressions for $\bar{P}^1$ and $\bar{P}^4$ it is immediate that this is so.

Truthful announcements, on the other hand, do not necessarily become more likely as ρ_M increases. Actually, the range of P_0 values that support a truthful announcement strategy starts by increasing in ρ_M but then decreases until only a knife-edge P_0 value supports a truthful announcement strategy when $\rho_M = 1$. This occurs because there are two opposing forces are at work when ρ_M increases. First, the difference between expected prices conditional on $s_M = H$ and $s_M = L$ tends to increase as the likelihood of $s = s_M$ increases. Second, the difference between these prices tends to decrease as investors focus more on the announcement instead of their signal when forming beliefs. When ρ_M is small relative to ρ, the first effect dominates, whereas when ρ_M becomes larger the second effect dominates. As discussed before, the difference between these prices is crucial to the existence of truthful announcement strategies. The larger it is, the larger the range of P_0 prices that support a truthful announcement strategy. Therefore, when ρ_M is small relative to ρ that range increases with ρ_M, but then at some point it starts to shrink as ρ_M increases.

One implication of this is that manipulation is more likely for large ρ_M than it is for small ρ_M. It is possible to show that the single price that supports a truthful announcement strategy when $\rho_M = 1$ is larger than the one when $\rho_M = 1/2$, provided that $\delta \leq 1$. Thus, upward manipulation is guaranteed to be more likely for large values of ρ_M, whereas downward manipulation may or may not be more likely to occur. In general, upward manipulation becomes the most likely announcement strategy when ρ_M is large relative to ρ.

The third part of the theorem 5 seems, at first sight, to indicate that tighter short sales constraints lead to more informative announcements, since the range of P_0 values for which there is a truthful or an informative announcement increase as δ decreases. However, this is only a second order effect, attributed to risk aversion. The main effect of a decrease in δ is that short positions become less attractive. As a result, the manipulator becomes more biased toward long positions. The implication is that both the upward manipulation, truthful announcement and downward manipulation regions shift toward larger values of P_0. Moreover, the region of upward manipulation widens, whereas the region of downward manipulation shrinks. Consequently, a smaller δ most likely decreases the average informativeness of the announcement and increases the probability that the announcement is manipulated upwards.

From the discussion above we can see that tying a relatively small δ to a large ρ_M relatively to ρ, upward manipulation most probably becomes the most likely announcement strategy among the informative ones. In such case, most of the times $a = L$ is a truthful signal for $s_M = L$, whereas $a = H$ is a manipulated signal which on average signals for $s_M = H$. This means that good information is spread through rumors (manipulated announcements) whereas bad information is spread through news (truthful announcements) which provides one possible explanation for the saying "buy on rumor, sell on news". Not only does this parametrization seem plausible, but also its implications are supported by Cliff's (2007) empirical finding that buy and hold recommendations of leading analysts (here the manipulator) appear to be less truthful than sell recommendations.

Corollary 2 *If δ is small and/or ρ_M is large relative to ρ, then it is very likely that upward manipulation is the most frequent announcement strategy. Hence, if prices are inelastic, investors buy on the rumor and sell on the news.*

Summing up the results obtained thus far, we have that the manipulator can be induced to make more informative announcements if: (i) the penalty for manipulation increases, provided that the regulator is skilled; and (ii) the quality of information available to investors from sources other than the manipulator increases, provided that the manipulator is himself well informed. In his study, Cliff (2007) obtains empirical evidence supporting these predictions. He finds that affiliated analysts are less likely to manipulate upward buy and hold recommendations after the regulatory changes introduced in 2002 and 2003 came into effect.

The regulatory changes encompass a mix of the two measures suggested above. On the one hand, NASD rule 2711 "Research Analysts and Research Reports" and NYSE rule 472 introduced regulation on the analyst's activity and the reports they disclose to the public. The increased scrutiny of analyst reports can be likened to an increase in k via an increase in the probability of uncovering manipulation. On the other hand, the 2003 Global Settlement between leading investment banks and regulators resulted in an increase of the quantity and quality of independent research. Investment banks are required to fund research by at least 3 government-approved independent analysts and distribute the independent reports alongside with their own research reports. This corresponds to an increase in ρ in the model, with all investors observing the report of the independent analyst.

Whether one measure or the other is more effective depends on the circumstances and on their cost of implementation. On the one hand, an increase in ρ is only effective if the manipulator is well informed. Otherwise, the frequency of manipulation is curtailed but at the expense of making the manipulator keep silent, instead of making the manipulator announce truthfully. In this case, manipulation is not bad, because the alternative is to not announce instead of announcing (more) truthfully. Rational investors can still extract useful information from a manipulated announcement, but not when the manipulator does not announce. Only if investors were naive, would it be preferable to have no announcement to a manipulated announcement.

On the other hand, an increase in k is highly effective but only if the manipulator can avoid punishment by not manipulating. If he is occasionally punished despite being truthful, an increase in the penalty may force him to not announce. The difficulty here is that, in practice, it is hard to uncover the manipulator's private information. Therefore, either penalties are few and far in between, and so the expected penalty is not large enough to induce truthfulness, or there is a fear of being punished without any wrongdoing, which may silence a cautious manipulator.

3.3 Price Efficiency, Risk Premium and Price Response to Announcement

In this subsection I look into the effect of the manipulator's announcements on price efficiency, risk premium and price response to the announcement. It should be quite obvious that all of the above are directly related to the informativeness of the manipulator's announcement. The more informative the manipulator's announcement, be it because it is more truthful or because the manipulator is better informed, the higher the price efficiency and the price response to the announcement, and the smaller the risk premium.

Price efficiency means, as usually in the literature (see e.g. Fishman and Hagerty, 1992), the amount of information incorporated in the equilibrium price. In this model, this corresponds to the information available to investors, since the manipulator's trades have no impact on the equilibrium price. Therefore, I use the probability investors assign to the true liquidation state, i.e. $p^I_{(H,a,-)}$ when V_H and $1-p^I_{(H,a,-)}$ when V_L occurs, as the measure of price efficiency. Of interest are the average price efficiency conditional on the informational state $(s,a,-)$, denoted by $\overline{eff}_{(s,a,-)}$, and the unconditional average efficiency, denoted by $\overline{eff}$, which are defined in appendix A.[18] Also in appendix A I show that in the simplified setting of this section, where all investors observe signal s for free, price efficiency and risk premium are inversely related.

The following theorem presents the impact of the manipulator's announcement strategy, ρ and ρ_M on price efficiency and risk premium.

[18] To be precise, the unconditional average efficiency is in fact conditional on the manipulator's announcement strategy.

Theorem 7 *Let $\widetilde{\theta}$ denote the frequency with which manipulation occurs. Then, when all investors observe signal s:*

(i) The unconditional average price efficiency (risk premium) increases (decreases) with the informativeness of the manipulator's announcement strategy, i.e., $\theta_{H|H} + \theta_{L|L}$;

(ii) All else equal, the unconditional average price efficiency (risk premium) increases (decreases) in ρ and, except in the NA equilibrium, in ρ_M;

(iii) For a given announcement a, the average price efficiency (risk premium) conditional on the informational states where s agrees with a is larger (smaller) than in the informational states where they disagree;

(iv) The average price efficiency (risk premium) conditional on the informational states where s and a agree always increases (decreases) in ρ and, except in the NA equilibrium, in ρ_M, and decreases (increases) in $\widetilde{\theta}$ in the equilibria where a is manipulated (UM equilibrium in the state $(H, H, -)$and DM equilibrium in the state $(L, L, -)$);

(v) The average price efficiency (risk premium) conditional on the informational states where s and a disagree decreases (increases) in ρ and increases (decreases) in ρ_M in the equilibria where a is not manipulated if and only if $\rho < \rho_M$; in the equilibrium where a is manipulated the above is true and, in addition, the average price efficiency (risk premium) increases in $\widetilde{\theta}$, if and only if $\widetilde{\theta} < \frac{1}{2}\frac{\rho_M - \rho}{\rho + \rho_M - 1}$ (that is, if and only if manipulation is not too frequent); when $\rho > \rho_M$ (which implies that $\widetilde{\theta} > \frac{1}{2}\frac{\rho_M - \rho}{\rho + \rho_M - 1}$) the opposite holds.

Proof. See Appendix B. ■

In the simplified setting of this section, the manipulator's announcement strategy has only one simple impact on the unconditional average price efficiency: the more informative the announcement strategy, the better informed each and every investor becomes, and so the higher the price efficiency.[19] Therefore, there is a one-to-one positive relation between the informativeness of the announcement strategy and price efficiency. Thus, given what we have seen in the previous subsection, the first two results should come with no surprise. However, I would like to emphasize that manipulation *per se* is not bad in terms of price efficiency

[19] As we will see in the general case, where investors have to decide whether to purchase signal s, the manipulator's announcement strategy will have an additional impact on price efficiency: in general, the more informative the announcement strategy, the less incentive there is to purchase the costly signal, which decreases price efficiency. Therefore, it is not obvious that the informativeness of the announcement strategy and price efficiency change in the same direction.

(point (i) of the theorem). This is only the case if the manipulator can be made to announce truthfully, and the first best is attained. If the alternative to manipulation is to not announce, then manipulation (the second best) is preferable. This has a clear policy implication: any measure designed to mitigate manipulation has to create the conditions necessary for the manipulator to announce (more) truthfully, and not make him go silent.

The last three results in the theorem show us that more information is not always good, because the two signals may disagree. In this case, investors know that one signal is right and the other is wrong; they just don't know which is what. This "confuses" investors and reduces the amount of information they can extract from the signals. In the limit where the two disagreeing signals are equally accurate, investors can't extract any information from them. It is then immediate that if both signals agree then investors are able to eliminate more uncertainty than if they disagree, and so the third point of the theorem follows. In addition, when both signals agree, any improvement in the quality of information received by investors (higher ρ or ρ_M, or less manipulation) reduces uncertainty. But the same is not true if signals disagree. In such a case, investors are less confused by the disagreeing signals if one of them is more accurate than the other. Hence, if the accuracy of the most accurate signal increases, investors extract more information from the signals and price efficiency increases while the risk premium decreases. If it is the accuracy of the least accurate signal that increases, the opposite happens. Nonetheless, averaging over the information states, better information undoubtedly contributes to higher price efficiency and a smaller risk premium.

To finalize the analysis of the simplified version of the model, I look at the average price response to the manipulator's announcement. Because the initial price is independent of the announcement, I focus only on the post-announcement price. On average, the post-announcement price when the announcement reveals good (bad) news will be larger (smaller) than the initial price and so the price response is larger the larger (smaller) the post-announcement price is.

Theorem 8 *In any equilibrium with informative announcement strategies the following holds:*

(i) $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \theta_{H|H}} > 0$, $\frac{\partial \bar{P}_{(s,L,-)}}{\partial \theta_{L|L}} < 0$, $\frac{\partial \left[\bar{P}_{(s,H,-)} - \bar{P}_{(s,L,-)}\right]}{\partial \theta_{H|H}} > 0$ *and* $\frac{\partial \left[\bar{P}_{(s,H,-)} - \bar{P}_{(s,L,-)}\right]}{\partial \theta_{L|L}} > 0$, $s = \{H, L\}$*;*

(ii) $\frac{\partial \bar{P}_{(s,H,-)}}{\partial \rho_M} > 0,\ \frac{\partial \bar{P}_{(s,L,-)}}{\partial \rho_M} < 0$ *and* $\frac{\partial[\bar{P}_{(s,H,-)} - \bar{P}_{(s,L,-)}]}{\partial \rho_M} > 0,\ s = \{H, L\};$

(iii) $\frac{\partial \bar{P}_{(H,a,-)}}{\partial \rho} > 0,\ \frac{\partial \bar{P}_{(L,a,-)}}{\partial \rho} < 0$ *and* $\frac{\partial[\bar{P}_{(H,a,-)} - \bar{P}_{(L,a,-)}]}{\partial \rho} > 0,\ a = \{H, N, L\}.$

Proof. See Appendix B. ■

In face of what we have seen until this point, these results are quite obvious. Prices are more sensitive to the manipulator's announcement the more truthful (point i) and the better informed (point ii) he is, regardless of s.[20] Similarly, prices are more sensitive to the investors' signal the more accurate it is (point iii). However, notice that if there is manipulation, prices react less to announcements that may have been manipulated (e.g. $a = H$ in the UM equilibrium) than to announcements that are known to be truthful (e.g. $a = L$ in the UM equilibrium). Summing up, the price reaction to the announcement is stronger the more uncertainty the announcement resolves.

An obvious consequence of a stronger initial price response to the announcement is that later on, when the liquidation value is revealed, the surprise is smaller. Also, since investors are rational, the post-announcement return will always be positive so that the residual risk is rewarded. The following corollary summarizes these results.

Corollary 3 *The average initial price response to the manipulator's announcement increases with the truthfulness of his announcement and with the accuracy of his information, while the average price response to the revelation of the liquidation value (the post-announcement return) decreases. The post-announcement return is always positive.*

These results suggest that we can test for the manipulation by looking at either the initial price response to the announcement, or to post announcement returns. However, the first method will always be more robust. First, it is a short window event, and so there is no need to compute normal returns. Second, it is much easier to attribute the price change to the announcement. Testing for manipulation using post-announcement returns requires the computation of risk-adjusted returns over the (subjectively determined) period of time we expect the event, on which the manipulator received a signal, to be observed or, at least, to be almost completely reflected in prices. The problem is that then one needs to control for all other information that

[20]Prices include the effect of both the announcement and the signal s. Setting $\rho = 1/2$ makes s irrelevant. However, the result holds for any $\rho < 1$.

is relevant to the asset's value but is not related to the event that underlies the announcement. Thus, in practice, detecting manipulation from post announcement returns is hardly feasible.

Detecting manipulation from the initial price response to the announcement, though, is not an easy feat either. If manipulators are not biased toward upward or downward manipulation, and so use both strategies equally likely, we will not be able to detect any bias in the price response to good or bad announcements. We will only observe a relatively small average price response to the announcement. But the same could be a result of a relatively small ρ_M, which is unobservable. These two scenarios could in principle be set apart from one another, since the former results in a higher volatility of price response to announcements (the price response to $a = L$ is large in an UM equilibrium but small in a DM equilibrium). However, this requires the existence of a benchmark to calibrate this volatility, which in practice is difficult to obtain, and assumes that ρ_M is fixed over time and across manipulators, which is hardly the case. In this case, the only feasible possibility is to take advantage of an event with predictable impact on manipulation, such as the regulation changes 2002/03.

If, on the other hand, there is a reason to believe that the manipulator is biased toward either upward or downward manipulation, it is easier to detect manipulation. In that case one should be able to detect a bias in the price response to good and bad announcements. Short selling constraints is the most obvious reason why such a bias (in this case toward upward manipulation) may exist.

Cliff (2007) uses both approaches to search for evidence of upward manipulation. In his study, analysts affiliated to investment banks in charge of securities issuance are suspected to have a bias toward upward manipulation because they want to please their customers. He finds that prices are more responsive to sell recommendations of affiliated analysts than to buy or hold recommendations before the 2002/03 regulation changes. However, price reactions to announcements of independent analysts, which are assumed to have no incentive to manipulate (or no bias toward upward of downward manipulation), do not show such asymmetry. Moreover, after the 2002/03 regulation changes, the asymmetry in the price reaction to buy recommendations of affiliated analysts increased, while the asymmetry between the reaction to buy and sell recommendations decreased. These empirical results are in line with the predictions of the model and suggest that (i) affiliated analysts, on average, manipulate their recommendations upward and (ii) that the 2002/03 regulation changes contributed to a decrease in that manipulation.

In turn, using the post announcement returns of portfolios formed based on the recommendations, Cliff finds that (i) all portfolios formed based on the recommendations of affiliated analysts underperform and (ii) all portfolios formed based on the recommendations of independent analysts have a neutral performance. He takes these results as evidence of upward manipulation by affiliated analysts and poorly informed but honest independent analysts. However, finding a negative abnormal return for the buy portfolio is not evidence of upward manipulation.[21] According to the model, the more truthful buy recommendations are, the smaller the post announcement return, since there is less residual uncertainty.

To correctly interpret his results we need to compare the relative performance of each portfolio, and not their absolute performance. This is because when the announcement is informative some uncertainty is resolved which results in a smaller post-announcement return. If the model for normal returns fails to capture this, negative risk-adjusted returns will result. Looking at the relative performance mitigates this problem. Proceeding in this way, what we observe is that buy and hold portfolios outperform sell portfolios, both in terms of raw returns and risk-adjusted returns. This asymmetry does suggest upward manipulation of buy and hold recommendations.

4 Equilibrium with Information Acquisition

In this section I drop the assumption that all investors observe signal s for free. Instead, they have to pay a cost c to observe s at date 1, being free to choose whether to purchase the information or not. This will lead to the existence of two types of investors: those who observe s (informed) and those who don't (uninformed). We can think that signal s comes from independent analysts who sell their recommendations, or that it is the result of research

[21] Cliff implicitly assumes that the underperformance of the buy portfolio results from investors realizing that they were fooled by the manipulated recommendation. However, on average if investors are rational and anticipate the manipulator's strategy they are not fooled by the manipulator. For a correction to exist on average (some) investors have to be naive. But if we assume that investors may not be rational, then the test for manipulation becomes a joint test for manipulation and investors' rationality. Besides, the conclusions taken from the reaction of prices to the announcement rely on the opposite assumption that investors are rational.

by investors which incur in time and monetary costs c.

The main point of interest of this section is in how the existence of a manipulator and his announcement strategy affects investors' choices of information acquisition. On one hand, the more informative the manipulator's announcement is, the less incentive there is to purchase the costly information. But, on the other hand, the smaller the fraction of informed investors, the more the announcement is manipulated, and so the less informative it is.

4.1 Solving for the Equilibrium

From the first order condition of the optimization problem 1 we obtain linear demands for the risky asset for both types of investors

$$X^I = \frac{p^I - P}{\alpha p^I \left(1 - p^I\right)},\ X^U = \frac{p^U - P}{\alpha p^U \left(1 - p^U\right)}.$$

Since there is a fraction λ, to be determined endogenously, of informed investors, the average per capita demand of the asset is given by $\lambda X^I + (1-\lambda) X^U$. Market clearing then implies that

$$P\left(s, a, \lambda, z\right) = \frac{\frac{\lambda}{1-p^I_{(s,a,-)}} + \frac{1-\lambda}{1-p^U_{(s,a,-;z)}} - \alpha z}{\frac{\lambda}{p^I_{(s,a,-)}\left[1-p^I_{(s,a,-)}\right]} + \frac{1-\lambda}{p^U_{(s,a,-;z)}\left[1-p^U_{(s,a,-;z)}\right]}},\ \forall s, a. \tag{3}$$

Uninformed investors form their conditional belief on the probability of V_H, $p^U_{(s,a,-;z)}$, using the information contained in the public announcement a and the equilibrium price to estimate the likelihood with which informed investors observed $s = H$, denoted by $\hat{\gamma}$. Uninformed investors know what informed investors beliefs would be if they observed $s = H$ or $s = L$. Thus, from the market clearing condition, they can figure out the only two possible pairs of informed investors' beliefs and asset supply that generate a given price, $\left(p^I_{(H,a,-)}, z_H\right)$ and $\left(p^I_{(L,a,-)}, z_L\right)$. That is, either informed investors observe good news ($s = H$) and the supply is relatively large, or they observe bad news ($s = L$) and the supply is relatively small.

Knowing the distribution of z, uninformed investors can then obtain the likelihood of observing z_H and z_L and estimate $\hat{\gamma}_{(s,a,-;z)}$, the probability with which $\left(p^I_{(H,a,-)}, z_H\right)$ occurred when the informational state is $(s, a, -)$ and random supply is z (all of this incorporated in the equilibrium price), as

$$\hat{\gamma}_{(s,a,-;z)} = \frac{\phi\left(z_H\right)}{\phi\left(z_H\right) + \phi\left(z_L\right)} \in (0,1)$$

where $\phi\left(.\right)$ if the density function for $z \sim \mathcal{N}\left(\bar{z}, \sigma_z^2\right)$. Notice that, because $\hat{\gamma}_{(s,a,-;z)}$ depends on the equilibrium price, there is the need to solve for a fixed point in the equilibrium price.[22] $p^U_{(s,a,-;z)}$ is then given by

$$p^U_{(s,a,-;z)} = \hat{\gamma}_{(s,a,-;z)} p^I_{(H,a,-)} + \left[1 - \hat{\gamma}_{(s,a,-;z)}\right] p^I_{(L,a,-)}, \forall s, a, z$$

where $p^I_{(s,a,-)}$ is exactly the same as in the equilibrium where all investors are informed (see appendix A).

From the equilibrium definition, given in Subsection 2.6, λ is such that no investor wants to change his information acquisition decision in equilibrium. This implies that, ex-ante, the expected utility of informed investors is the same, larger and smaller than that of uninformed investors if $\lambda \in (0,1)$, $\lambda = 1$ and $\lambda = 0$, respectively.

The manipulator chooses his optimal announcement and trading strategy exactly in the same way as in the previous section, when all investors were informed. Once again, the equilibrium has to be solved for numerically using McKelvey's (1992) algorithm.

One thing I would like to note is that it is always possible that $\hat{\gamma}_{(H,a,-;z)} < 1/2$ (or $\hat{\gamma}_{(L,a,-;z)} > 1/2$), that is, uninformed investors think that $a = L$ ($a = H$) is more likely to have occurred when in fact $a = H$ ($a = L$) occurred. However, on average uninformed investors get it right.

Looking at expression (3) for the equilibrium price, we can see that it has the same linear form as in the case where all investors were informed, expression (2). Comparing expressions

[22] The only case where one can avoid solving for the fixed point is when informed investors have the same belief on the variance of V in the informational states $(H, a, -)$ and $(L, a, -)$. In this case, $\hat{\gamma}_{(s,a,-;z)}$ can be expressed in terms of the primitives of the model, and there is no explicit dependence on P. If the manipulator does not announce, this is always the case. But if he announces, unfortunately, we have to solve for the fixed point.

(2) and (3) we can obtain the beliefs of the representative investor as being

$$\begin{aligned}
E\left(V \mid s,a\right) &= \frac{\frac{\lambda}{1-p^{I}_{(s,a,-)}} + \frac{1-\lambda}{1-p^{U}_{(s,a,-;z)}}}{\frac{\lambda}{p^{I}_{(s,a,-)}\left[1-p^{I}_{(s,a,-)}\right]} + \frac{1-\lambda}{p^{U}_{(s,a,-;z)}\left[1-p^{U}_{(s,a,-;z)}\right]}} \\
Var\left(V \mid s,a\right) &= \frac{1}{\frac{\lambda}{p^{I}_{(s,a,-)}\left[1-p^{I}_{(s,a,-)}\right]} + \frac{1-\lambda}{p^{U}_{(s,a,-;z)}\left[1-p^{U}_{(s,a,-;z)}\right]}}
\end{aligned}$$

which are weighted averages of the beliefs of informed and uninformed investors. $p^{R}_{(s,a,-)}$, the probability the representative investor assigns to V_H, can then be obtained from $p^{R}_{(s,a,-)} = E\left(V \mid s,a\right)$. Another alternative is to use $p^{R}_{(s,a,-)}\left(1-p^{R}_{(s,a,-)}\right) = Var\left(V \mid s,a\right)$ to obtain $p^{R}_{(s,a,-)}$. These two alternatives do not yield the same $p^{R}_{(s,a,-)}$, although they are very close. In the next subsections, the results on price efficiency are based on $p^{R}_{(s,a,-)} = E\left(V \mid s,a\right)$.

4.2 The Manipulator's Optimal Strategy

Dropping the assumption that all investors observe signal s has no direct impact on the manipulator's announcement and trading strategies analyzed in the previous section. The only impact is indirectly through a decrease in the amount of information available to investors that is obtained from signal s. When less investors are informed the impact of signal s on the equilibrium price is then naturally smaller, since uninformed investors can only extract a weaker version of the signal from the equilibrium price. From the point of view of the manipulator, a smaller fraction of informed investors is qualitatively the same as a decrease in ρ, the accuracy of signal s, when all investors are informed. All that matters to the choice of the announcement strategy is how sensitive equilibrium prices are to signal s. The more sensitive they are, be it because more investors observe the signal or the signal is more accurate, the less room for manipulation there is (see figure 2). Therefore, all the analysis on the manipulator's announcement strategy performed in the previous section is still valid here.

4.3 Investors' Acquisition of Information

The next theorem provides the results on the optimal fraction of informed investors. Because the beliefs of informed investors cannot be obtained in closed form solution, these results come from extensive numerical simulations using different parameters. Therefore, they cannot be

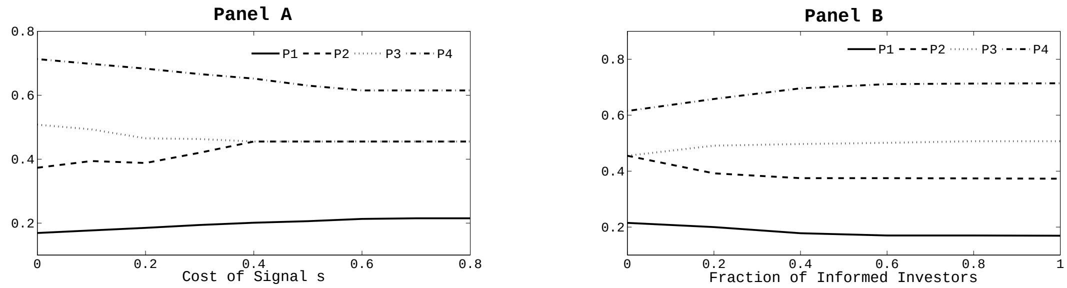

Figure 2: Impact of the fraction of informed investors on the manipulator's announcement strategy. Panel A shows the impact of the cost of signal s on the thresholds for each type of announcement strategy, indirectly through its impact on the fraction of informed investors. Panel B shows the direct impact of the fraction of informed investors (higher c implies a smaller λ). In both panels the parametrization used was: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\rho = 0.8$, $\delta = 1$, $k = 0$, $\rho_M = 0.9$. The integration over z, necessary to compute the equilibria, was performed via Gauss-Hermite quadrature with 15 nodes.

proved in full generality. For this reason the theorem includes the qualifier "in general".

Theorem 9 *In general, the optimal fraction of informed investors, λ, satisfies the following properties:*

(i) λ decreases in the cost of information, c, with $\lambda = 1$ if $c = 0$ and $\lim_{c \to +\infty} \lambda = 0$;

(ii) $\lambda = 0$ for any c if the manipulator announces truthfully and his information is perfectly accurate, i.e., $\rho_M = 1$; in any other case, $\lambda > 0$ provided that c is low enough;

(iii) the existence of a manipulator decreases λ, but only if the manipulator's announcement is informative enough, i.e. ρ_M is large and/or the manipulator manipulates infrequently, and the equilibrium price is not very informative, i.e. ρ is small, c is large and /or σ_z^2 is large; otherwise, the opposite happens.

The first result is very intuitive and is the reflex of two things. First, putting the cost aside, being informed is always desirable to being uninformed because informed investors use their informational advantage over uninformed investors to take better trading decisions and obtain higher expected utility than the latter. Second, the information about signal s that uninformed investors extract from the equilibrium price is never a perfect substitute for the observation of the actual signal. Therefore, if the observation of s is costless, all investors choose to observe s.

The second point is also rather intuitive. If the manipulator's announcement is truthful and his information is perfectly accurate, then the observation of s cannot provide any additional information. Therefore, no one is willing to pay a strictly positive cost to observe the redundant information. From theorem 5 we know that this event is possible although it has zero-measure. In any other circumstances, s provides additional information which, for the right price, some investors are willing to purchase.

The third and final result is the most interesting and surprising one. A priori one would expect investors to substitute the costly information for the partially informative announcement to some extent. And most of the times this is exactly what happens. But not always. To understand the impact of the manipulator's announcement on λ, we need to look at its impact on the informed and uninformed investors' beliefs. (I focus here on beliefs to simplify the exposition, loosely assuming that differences in beliefs are one-to-one with differences in expected utility, which is what determines λ.) The announcement, being public, provides more information to all investors. However, it does not benefit all investors equally. Prior to any adjustment in λ, the announcement may increase or decrease the informational advantage of informed investors over uninformed investors. In the first case, some uninformed investors will find optimal to pay the cost and become informed, thus increasing λ, whereas in the second case some informed investors will no longer find it optimal to pay the cost and choose to become uninformed, thus decreasing λ.

Most of the times, the announcement ends up benefiting more uninformed investors than informed investors. This happens because the incremental information content of the announcement is likely to be larger for uninformed investors than for informed investors, since the latter are already better informed. It is easy to see this point in the limit case where informed investors are perfectly informed and uninformed investors are completely uninformed. In this case the announcement has no incremental information for informed investors, but it provides information for uninformed investors, decreasing the informational disadvantage of being uninformed.

However, there is another effect at play here: the interaction between sources of information. Whereas the announcement has no impact on the signal s observed by informed investors, it does have an impact on the information about s that uninformed investors extract from the equilibrium price. If informed investors observe $s = H$, they will not put into question that they observed $s = H$ if they observe $a = L$. They will only question the veracity of $s = H$, but not that they observed $s = H$. On the other hand, if uninformed investors think, from the observation of the equilibrium price, that there is a 75% chance that $s = H$ when there is no announcement, the observation of $a = L$ will lead (most of the times) to a downward revision of that probability. Thus, the announcement has a negative impact on the information that uninformed investors extract from the equilibrium price, which in most cases translates into a negative impact on the expected utility on uninformed investors.

Because the two effects described above have the opposite sign, we need two things to happen for λ to increase due to the announcement. First, uninformed investors have to extract a significant amount of information from the equilibrium price, so that the second effect is strong. This means that the accuracy of s (ρ) should be large and the cost of s (c) and the supply noise (σ_z^2) small. Second, the informativeness of the manipulator's announcement (function of ρ_M and the frequency of manipulation) must not be too large, so that the first effect is relatively weak.

Figure 3 illustrates this. When $\rho_M = 1/2$, announcements, even if truthful, are uninformative, and so it is the same as if no announcement is made. Thus, the value of λ when $\rho_M = 1/2$ is the no announcement benchmark fraction of informed investors. The figure shows that announcements can increase λ if ρ_M is not too large and ρ is large (panel A), c is small (panel B) and σ_z^2 is small (panel C). In panel D we can see that more manipulation has a similar effect to a smaller ρ_M.

In the exposition above, it was assumed that differences in beliefs are one-to-one with differences in expected utility. Although they are strongly associated, there is another important factor to take in consideration: the expected utility obtained in equilibrium tends to decrease as the residual uncertainty decreases.[23] This implies that there are circumstances where the announcement increases the gap in the beliefs of informed and uninformed investors, but actually decreases the gap in their expected utilities, since both converge to zero. Because the cost of information is fixed, this results in a decrease in λ. Therefore, the odds are against

[23] Although this may seem strange, it is easy to see that this is true. If there is residual uncertainty, investors will only hold the risky asset if they obtain a positive expected utility, since they can obtain a zero expected utility from the risk free asset. But when there is no residual uncertainty, the risky asset is actually risk free, and investors obtain a zero expected utility. This is not a feature exclusive of this model. Any normal-CARA model, commonly used in the literature for its tractability (e.g. Grossman and Stiglitz, 1980), produces the same results because of the implied mean-variance preferences, which I assumed explicitly in this paper.

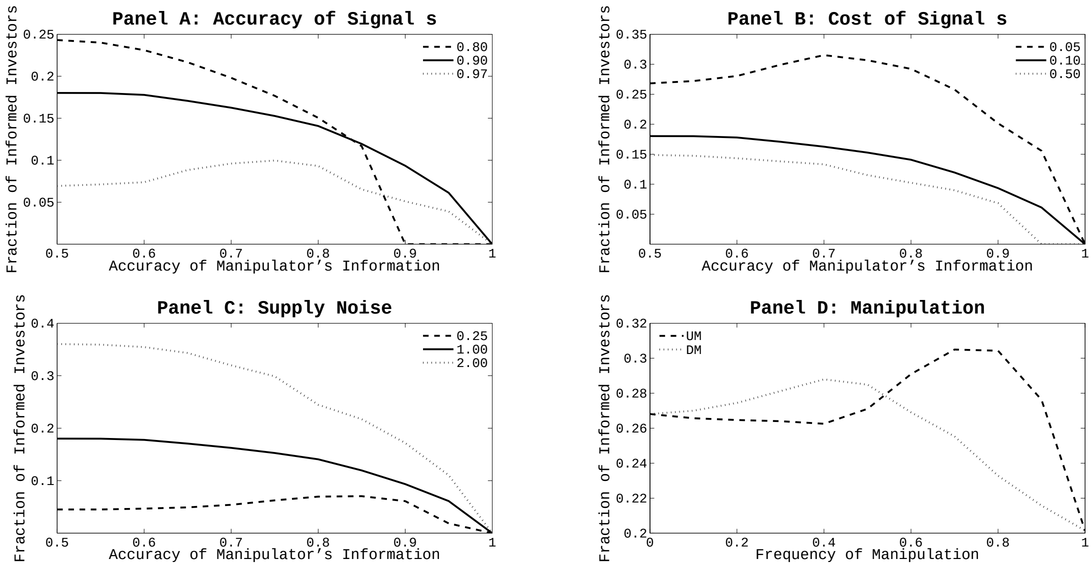

Figure 3: Impact of announcements on the fraction of informed investors. Panels A to C present the impact of a truthful announcement strategy on the fraction of informed investors (λ) when the accuracy of the manipulator's information (ρ_M) ranges from 0.5 to 1. When $\rho_M = 0.5$ announcements are uninformative and it is the same as if no announcement is made. In panels A to C the filled line corresponds to the following base parametrization: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\rho = 0.9$, $c = 0.3$,$\delta = 1$, $k = 0$. In panel A the dashed line corresponds to $\rho = 0.8$ and the dotted line to $\rho = 0.97$; in panel B the dashed line represents $c = 0.05$ and the dotted line $c = 0.5$; in panel C the dashed line represents $\sigma_z = 0.25$ and the dotted line $\sigma_z = 2$. Panel D presents the impact of manipulation on the λ. The parametrization is the same as in the dashed line of panel B (base case with $c = 0.05$) and $\rho_M = 0.9$. The dashed line corresponds to the case of upward manipulation and the dotted line to the case of downward manipulation. The integration over z, necessary to compute the equilibria, was performed via Gauss-Hermite quadrature with 15 nodes.

finding an increase in λ as a response to the announcement.

One final thing I would like to notice is that, although an increase in λ does contribute to an improvement in truthfulness of the manipulator's announcement strategy and thus of its informativeness, this is not one motivation for the increase in λ. This is because investors are atomistic and so their individual decisions on whether to become informed or not has no impact on the manipulator's announcement strategy. However, even in a setting where investors are not atomistic, and thus and take in consideration the impact of their information acquisition decisions on the announcement strategy, they would not purchase more information to boost the information content of the announcements. All investors would benefit from the more informative announcements, but those who bear the entire cost associated to the additional informativeness of the announcement are the ones who, in general, benefit less from it.

4.4 Price Efficiency and Risk Premium

We have just seen that announcements may benefit (in terms of expected utility) more informed investors than uninformed investors, thus inducing an increase in λ. However, this does not necessarily mean that the beliefs of uninformed investors, and consequently those of the representative investor, deteriorate because of the announcement. First of all, beliefs and expected utility, although associated with one another, are not one-to-one. Second, even if they were, all that is necessary for λ to increase is that, prior to the adjustment in λ, informed investors benefit more than uninformed investors from the announcement, not that the latter are made worse-off by the announcement. Third, after the increase of λ in response to the announcement, the remaining uninformed investors can extract more information from prices because there are more informed investors. Thus, all else equal, their beliefs improve, which may be enough to compensate for the deterioration induced by the announcement.

Thus, to address the question of whether equilibrium beliefs deteriorate in response to the announcement, I analyze the impact of announcements on: (i) the average probability that informed, uninformed and the representative investor assign to the true liquidation state; (ii) the expected risk premium for the representative investor. The average probability that the representative investor assigns to the true liquidation state is used as the measure of price efficiency.

Theorem 10 *In general, the following holds:*

(i) announcements increase the average probability that informed investors assign to the true liquidation state;

(ii) announcements increase the average probability that uninformed investors and the representative investor assign to the true liquidation state but only if ρ_M is large and/or the manipulator manipulates infrequently, ρ is small, c is large or small and /or σ_z^2 is large or small; otherwise, the opposite happens.

As expected, based on the results in section 3 where all investors are informed, the beliefs of informed investors improve with the extra information obtained from the announcement. But the beliefs of uninformed investors may indeed deteriorate due to the announcement. As a

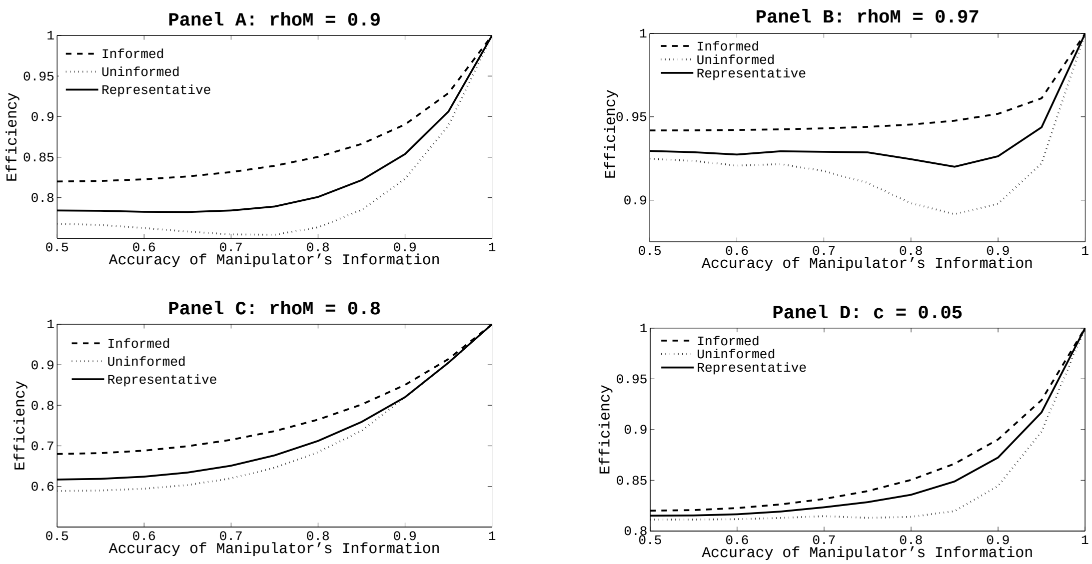

Figure 4: Impact of announcements on price efficiency. All panels present the impact of a truthful announcement strategy on the average probability that informed, uninformed and the representative investor assign to the true liquidation state. The latter correspond to the measure of price efficiency. In all panels the parametrization is: $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\delta = 1$, $k = 0$. In addition, $\rho_M = 0.9$, $\rho_M = 0.97$ and $\rho_M = 0.8$ in panels A, B and C, respectively, $c = 0.3$ in panels A to C and $c = 0.05$ in panel D.

consequence, price efficiency may decrease if the fraction of uninformed investors is relatively large.

Figure 4 shows how the average probability that each type of investor assigns to the true liquidation state reacts to truthful announcements for different values of ρ_M and for $\rho = 0.9$ (panel A), $\rho = 0.97$ (panel B) and $\rho = 0.8$ (panel C).

Comparing with panel A of figure 3 we can see that increases in the gap between the beliefs of informed and uninformed investors do not necessarily imply increases in λ (compare panel A of figure 3 with panels A and B of figure 4). This happens because at the same time that the announcement increases the gap in beliefs, it decreases the residual uncertainty. The former tends to increase the gap in expected utility, whereas the latter tend to decrease it. This means that the larger ρ_M, the larger the gap in beliefs that is necessary to induce an increase in λ.

The converse is also true: increases in λ do not necessarily imply an increase in the gap between beliefs of informed and uninformed investors (compare panel B of figure 3 with panel D of figure 4). This happens when beliefs of uninformed investors improve but less than those of informed investors. This is the case when for instance c or σ_z^2 are very small. In both

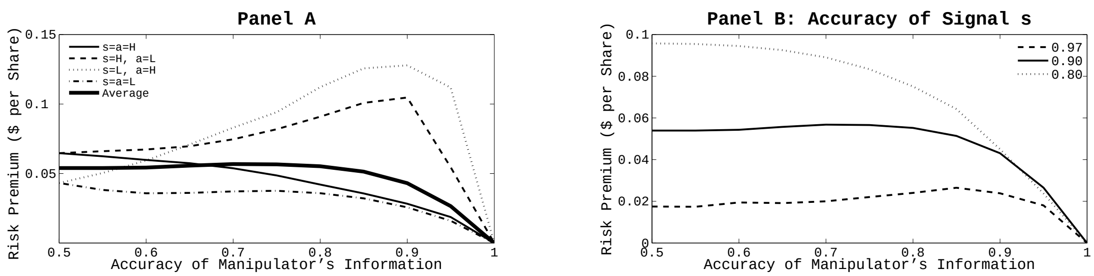

Figure 5: Impact of announcements on risk premium. Panel A plots the expected risk premium conditional on each of the 4 informational states observed by investors, and the unconditional expected risk premium, when the announcement is truthful. Panel B plots the unconditional expected risk premium for 3 values of ρ: 0.9 (filled line), 0.97 (dashed line) and 0.8 (dotted line). In all panels $\alpha = 0.5$, $\bar{z} = 1$, $\sigma_z = 1$, $\rho = 0.9$, $c = 0.3$,$\delta = 1$, $k = 0$, and in panel A $\rho = 0.9$.

cases, investors extract lots of information from prices. If, on the one hand, this implies that announcement has a negative impact on a source of information that is heavily weighted by uninformed investors, on the other hand they are less "confused" by the announcement which mitigates its negative impact.

Theorem 11 *In general the following holds:*

(i) announcements decrease the expected risk premium conditional on s and a agreeing;

(ii) announcements increase the expected risk premium conditional on s and a disagreeing, except when a is considerably more informative than s;

(iii) announcements decrease the unconditional expected risk premium but only if ρ_M is large and/or the manipulator manipulates infrequently, ρ is small, c is large or small and /or σ_z^2 is large or small; otherwise, the opposite happens.

The first two results of the theorem are similar to those on theorem 7, when it was assumed that all investors observed s. It reflects the fact that disagreeing signals confuse investors, which don't know which signals is correct, whereas agreeing signals reinforces the belief that both signals are correct. The third result tells us that the expected risk premium and price efficiency behave similarly. However, this does not necessarily mean that when announcements deteriorate price efficiency the expected risk premium increases, or vice-versa. Figure 5 provides an illustration of the impact of announcements on the expected risk premium.

To sum up, although most of the times announcements contribute to improved price efficiency and lower expected risk premium, this is not guaranteed to happen. Announcements, even if truthful, confuse investors whenever they disagree with the other sources of information available to investors. Despite this, on average they improve the beliefs of informed investors, because more often than not they agree with the other sources of information and reinforce the belief that the information is correct. However, the same may not be true for uninformed investors because they are more prone to confusion. This is so because uninformed investors have to infer the signal s that only informed investors observed, and announcements interfere with that. Notice that this problem is present even in the absence of manipulation. Hence, this problem is not exclusive of the manipulator's announcements, and can happen for any source of information. However, manipulation makes the problem more likely to occur.

4.5 Equilibrium in the Sequential Model

In the previous subsections it was assumed that the investors' decision to purchase information and the manipulator's announcement occur simultaneously. Here, I will briefly discuss what would change if investors take their decision only after observing the announcement.

Consider that the manipulator manipulates his announcement upwards (the case of downward manipulation is very similar). As we have seen in section 3, in this case the manipulator only announces $a = L$ when $s_M = L$, whereas he may announce $a = H$ when either $s_M = H$ or $s_M = L$. Thus, observing $a = L$ is more informative than observing $a = H$. As a result, the signal s provides less information in addition to that of the announcement when $a = L$. Therefore, when investors can condition their decision to become informed on the announcement, less investors will choose to become informed when $a = L$ than when $a = H$. Moreover, the fraction of informed investors is larger (smaller) when $a = H$ ($a = L$) than what it would be if investors could not condition their decision on the announcement.

The smaller fraction of informed investors when $a = L$ creates an incentive to manipulate upward whereas the larger fraction of informed investors when $a = H$ creates the opposite incentive. In general, the second effect dominates and so upward manipulation is less frequent in the sequential model. However, the same does not happen when the announcement is manipulated downward. In that case manipulation tends to increase in the sequential model. The reason for this asymmetry is that the fraction of informed investors when the manipulator announces truthfully is larger when $a = H$ than when $a = L$. Therefore, although upward (downward) manipulation favors a larger fraction of informed investors when $a = H$ ($a = L$), there is always a bias toward a larger fraction of informed investors when $a = H$. In turn, this bias contributes to less upward manipulation but to more downward manipulation.

5 Conclusion

In the introduction I raised a few questions I tried to answer throughout the paper. It's worthwhile to conclude the paper with a summarized answer to those questions.

Is the presence of the manipulator welcomed or not? If all investors are informed, yes, even if he manipulates. Rational and informed investors are able to make the right inference and extract useful information even when there is manipulation. However, uninformed investors may, in certain circumstances, be confused by the extra information, which makes the presence of the manipulator undesirable. But that is true for any other source of information.

Are the regulation changes of 2002/03 effective in mitigating manipulation? In most circumstances, yes. But if the manipulator is not sufficiently well informed, making investors better informed may increase manipulation in some rare cases.

Are there any undesirable side effects? Yes. The manipulator may be forced to not announcing instead of being forced to announce more truthfully. In the first case, price efficiency decreases and the risk premium increases.

Will the manipulator be pushed away from more regulated markets/assets to less regulated markets/assets? Depends on how the penalties for manipulation are implemented. If the manipulator can avoid being punished by announcing truthfully, he will prefer to announce in more regulated market, because they help him commit to announce truthfully. However, if he is punished based on discrepancies between what he announces and what the liquidation value is, he cannot completely avoid penalties by being truthful and may be forced to move to less regulated markets.

Appendix

A Solving for the Equilibrium without Information Acquisition

Here I provide more details on how to compute the equilibrium when all investors are informed. The probability investors assign to V_H given the signal s and announcement a they observe, $p^I_{(s,a,-)}$, is straightforward to obtain from the event tree associated to the information environment described in Section 2.

$$\begin{aligned} p^I_{(H,a,-)} &= \rho \frac{\theta_{a|H}\rho_M + \theta_{a|L}(1-\rho_M)}{2\gamma_{(H,a,-)}}, \ a \in \{H, L, N\} \\ p^I_{(L,a,-)} &= (1-\rho)\frac{\theta_{a|H}\rho_M + \theta_{a|L}(1-\rho_M)}{2\gamma_{(L,a,-)}}, \ a \in \{H, L, N\} \end{aligned}$$

where

$$\begin{aligned} \gamma_{(H,a,-)} &= \frac{\rho\left[\theta_{a|H}\rho_M + \theta_{a|L}(1-\rho_M)\right] + (1-\rho)\left[\theta_{a|H}(1-\rho_M) + \theta_{a|L}\rho_M\right]}{2}, \ a \in \{H, L, N\} \\ \gamma_{(L,a,-)} &= \frac{\rho\left[\theta_{a|L}\rho_M + \theta_{a|H}(1-\rho_M)\right] + (1-\rho)\left[\theta_{a|L}(1-\rho_M) + \theta_{a|H}\rho_M\right]}{2}, \ a \in \{H, L, N\} \end{aligned}$$

denote the probability of the information state $(s, a, -)$.

With these probabilities in hand, we can use the expression (2) to obtain the expected equilibrium price in the information scenario $(s, a, -)$,

$$\bar{P}_{(s,a,-)} \equiv E(P \mid s, a) = p^I_{(s,a,-)} - p^I_{(s,a,-)}\left[1 - p^I_{(s,a,-)}\right]\alpha\bar{z}.$$

The expected price in the information state $(-, a, s_M)$, necessary to compute the manipulator's expected utility, is obtained from $\bar{P}_{(s,a,-)}$ as follows:

$$\begin{aligned} \bar{P}_{(-,a,s_M)} \equiv E(P \mid a, s_M) &= Prob(s = H \mid s_M)\,\bar{P}_{(H,a,-)} + [1 - Prob(s = H \mid s_M)]\,\bar{P}_{(L,a,-)} \\ Prob(s = H \mid s_M = H) &= \rho\rho_M + (1-\rho)(1-\rho_M) \geq 1/2 \\ Prob(s = H \mid s_M = L) &= 1 - Prob(s = H \mid s_M = H) \leq 1/2. \end{aligned}$$

Taking the derivative of $\bar{P}_{(s,a,-)}$ with respect to $p^I_{(s,a,-)}$ and evaluating it at $p^I_{(s,a,-)} = 0$, we obtain that

$$\left.\frac{\partial \bar{P}_{(s,a,-)}}{\partial p^I_{(s,a,-)}}\right|_{p^I_{(s,a,-)}=0} = 1 + \left(2p^I_{(s,a,-)} - 1\right)\alpha\bar{z} = 1 - \alpha\bar{z}. \tag{4}$$

It is then clear that if the average amount of risk ($\bar{z}$) and risk aversion (α) are large enough so that $\alpha\bar{z} > 1$, then investors would pay more for a sure value of V_L than for a lottery which pays V_H or V_L with positive probability. To rule out this behavior from showing up on average prices, I make the following assumption.

Assumption 1 *The level of risk aversion and the average amount of risk are moderate. i.e.,* $0 \leq \alpha\bar{z} \leq 1$.

Therefore, the derivative of $\bar{P}_{(s,a,-)}$ with respect to all parameters except α and $\bar{z}$ are a positive linear function of the derivative of $p^I_{(s,a,-)}$ with respect to those same parameters, which is readily obtained.

The average price efficiency conditional on informational state $(s, a, -)$, denoted by $\overline{eff}_{(s,a,-)}$, is the probability investors assign to V_H, $p^I_{(s,a,-)}$, when V_H occurs, which happens with probability $p^I_{(s,a,-)}$, plus the probability they assign to V_L, $1 - p^I_{(s,a,-)}$, when V_L occurs, which happens with probability $1 - p^I_{(s,a,-)}$. The unconditional average price efficiency, denoted by $\overline{eff}$ is obtained by multiplying each $\overline{eff}_{(s,a,-)}$ by the probability of the respective informational state, $\gamma_{(s,a,-)}$, which is a function of the manipulator's announcement strategy. When all investors are informed, $\overline{eff}$ and $\overline{eff}_{(s,a,-)}$ are given by

$$\begin{aligned}\overline{eff}_{(s,a,-)} &= \left[p^I_{(s,a,-)}\right]^2 + \left[1 - p^I_{(s,a,-)}\right]^2 \\ \overline{eff} &= \sum_{s,a} \overline{eff}_{(s,a,-)}\gamma_{(s,a,-)}.\end{aligned}$$

The expected per share risk premium conditional on $(s, a, -)$, $\overline{RP}_{(s,a,-)}$, and the unconditional expected risk premium are given by

$$\begin{aligned}\overline{RP}_{(s,a,-)} &\equiv E\left(P - V \mid s, a\right) = p^I_{(s,a,-)}\left[1 - p^I_{(s,a,-)}\right]\alpha\bar{z} \\ \overline{RP} &= \sum_{s,a} \overline{RP}_{(s,a,-)}\gamma_{(s,a,-)}.\end{aligned}$$

It is straightforward to see that price efficiency and risk premium are inversely related

$$\overline{RP}_{(s,a,-)} = \frac{1 - \overline{eff}_{(s,a,-)}}{2}\alpha\bar{z}.$$

B Proofs

Lemma 1 *Under assumption 1, when all investors observe the signal s ($\lambda = 1$), the following holds for $s \in \{H, L\}$, $a \in \{H, L, N\}$ and $s_M \in \{H, L\}$:*

(i) $\bar{P}_{(H,a,-)} > \bar{P}_{(L,a,-)}, \bar{P}_{(-,a,H)} > \bar{P}_{(-,a,L)}$;

(ii) $\bar{P}_{(s,H,-)} > (=)\,[<]\,\bar{P}_{(s,N,-)} \wedge \bar{P}_{(-,H,s_M)} > (=)\,[<]\,\bar{P}_{(-,N,s_M)},\ if\ \frac{\theta_{H|H}}{\theta_{H|L}} > (=)\,[<]\,\frac{\theta_{N|H}}{\theta_{N|L}}$;

(iii) $\bar{P}_{(s,N,-)} > (=)\,[<]\,\bar{P}_{(s,L,-)} \wedge \bar{P}_{(-,N,s_M)} > (=)\,[<]\,\bar{P}_{(-,L,s_M)},\ if\ \frac{\theta_{N|H}}{\theta_{N|L}} > (=)\,[<]\,\frac{\theta_{L|H}}{\theta_{L|L}}$;

(iv) $\begin{cases} \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} > 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|H}} > 0 & if\ \theta_{a|L} > 0 \wedge 1/2 < \rho < 1 \wedge \rho_M > 1/2 \\ \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|H}} = 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|H}} = 0 & otherwise \end{cases}$ *and*

$\begin{cases} \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|L}} < 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|L}} > 0 & if\ \theta_{a|H} > 0 \wedge 1/2 < \rho < 1 \wedge \rho_M > 1/2 \\ \frac{\partial \bar{P}_{(s,a,-)}}{\partial \theta_{a|L}} = 0 \wedge \frac{\partial \bar{P}_{(-,a,s_M)}}{\partial \theta_{a|L}} = 0 & otherwise \end{cases}$;

(v) $\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} > (=)\,\bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}\ if\ \bar{z} > (=)\,0.$

If one of the ratios $\frac{\theta_{a|H}}{\theta_{a|L}}$, $a \in \{H, L, N\}$ is undefined because a is a zero-probability event, then it is considered to be equal to 1 (equally likely trembles).

Proof. The proof involves only a lot of algebra and so is omitted. It is available at the reader's request. ■

The conditions on the announcement strategy in (ii) and (iii) of the lemma above simply mean that

$a = H$ is a better signal for $s_M = H$ than $a = N$ (it is more likely that $s_M = H$ when we observe $a = H$ than when $a = N$), and $a = L$ is a better signal for $s_M = L$ than $a = N$, respectively.

Proof of theorem 1. The proof will proceed as follows. First I show that the strategies for each of the 5 subsets of P_0 are one equilibrium. Then, I characterize all other equilibria for each of those subsets and show that the equilibrium of theorem 1 is the focal one.

Truthful announcement (TA): $P_0 \in \left[\bar{P}^2, \bar{P}^3\right]$

If $\theta_{H|H} = \theta_{L|L} = 1$, then from parts (ii) and (iii) of lemma 1 we have

$$\bar{P}_{(-,H,s_M)} > \bar{P}_{(-,N,s_M)} > \bar{P}_{(-,L,s_M)},\ s_M \in \{H, L\}, \tag{5}$$

since $\frac{\theta_{H|H}}{\theta_{H|L}} = \infty > \frac{\theta_{N|H}}{\theta_{N|L}} = 1 > \frac{\theta_{L|H}}{\theta_{L|L}} = 0$.

At any information scenario $(-, a, s_M)$, the manipulator's trading strategy $\mathcal{T}_{(-,a,s_M)}$ is determined as

$$\begin{cases} \mathcal{T}_{(-,a,s_M)} = 1 & if\ \bar{P}_{(-,a,s_M)} - P_0 \geq \delta\left(P_0 - \bar{P}_{(-,a,s_M)}\right) \\ \mathcal{T}_{(-,a,s_M)} = -\delta & otherwise \end{cases}.$$

Suppose then that $\mathcal{T}_{(-,H,H)} = -\delta$. This means that $\bar{P}_{(-,H,H)} - P_0 < \delta\left(P_0 - \bar{P}_{(-,H,H)}\right)$ and consequently $P_0 > \bar{P}_{(-,H,H)}$. In this case $\Pi_{(-,L,H)} > \Pi_{(-,H,H)}$ since $\bar{P}_{(-,H,H)} > \bar{P}_{(-,L,H)}$ from (6) and so the manipulator deviates from $\theta_{H|H} = 1$. Therefore, $\mathcal{T}_{(-,H,H)} = -\delta$ cannot hold in a truthful equilibrium.

It must then be the case that $\mathcal{T}_{(-,H,H)} = 1$, which implies that $P_0 \leq \bar{P}_{(-,H,H)}$. Then, for $\theta_{H|H} = 1$ to be optimal, we must have $\Pi_{(-,H,H)} \geq \Pi_{(-,L,H)}$ and $\Pi_{(-,H,H)} \geq \Pi_{(-,N,H)}$. It is straightforward to see that this holds whenever $\bar{P}_{(-,H,H)} - P_0 \geq \delta\left(P_0 - \bar{P}_{(-,L,H)}\right)$. Proceeding in the same way, $\theta_{L|L} = 1$ requires that $P_0 > \bar{P}_{(-,L,L)}$ and $\delta\left(P_0 - \bar{P}_{(-,L,L)}\right) \geq \bar{P}_{(-,H,L)} - P_0$. Bringing together both conditions and noting that here $\bar{P}_{(-,a,s_M)} = \bar{P}^T_{(-,a,s_M)}$, $\theta_{H|H} = \theta_{L|L} = 1$ is optimal whenever

$$\bar{P}^2 \equiv \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta} \leq P_0 \leq \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta} \equiv \bar{P}^3.$$

Since from part (i) of lemma 1 $\bar{P}_{(-,a,H)} \geq \bar{P}_{(-,a,L)}$, the above condition can be satisfied. The associated trading strategy is $\mathcal{T}_{(-,H,H)} = 1$ and $\mathcal{T}_{(-,L,L)} = -\delta$.

Upward manipulation (UM): $P_0 \in \left(\bar{P}^1, \bar{P}^2\right)$

Suppose that $P_0 < \bar{P}^2$ marginally. Then from what we have seen above, the manipulator will want to deviate from $\theta_{L|L} = 1$, but not from $\theta_{H|H} = 1$. Suppose he deviates from $\theta_{L|L}$ to $\theta_{H|L}$.[24] Since $\theta_{H|H} = 1$, from part (iv) of lemma 1 $\bar{P}_{(-,L,H)}$ and $\bar{P}_{(-,L,L)}$ remain unchanged whereas $\bar{P}_{(-,H,H)}$ and $\bar{P}_{(-,H,L)}$ decrease. This implies that $\frac{\bar{P}_{(-,H,L)} + \delta \bar{P}_{(-,L,L)}}{1+\delta}$ (and $\frac{\bar{P}_{(-,H,H)} + \delta \bar{P}_{(-,L,H)}}{1+\delta}$ as well) decreases. Therefore, at some point

$$\frac{\bar{P}_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta} = P_0 \leq \frac{\bar{P}_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta}$$

[24] Deviation to $\theta_{N|L}$ will make $a = N$ and $a = L$ substitute signals for $s_M = L$ and will not lead to an equilibrium.

and $\theta_{H|H} = 1$, $\theta_{H|L} = \omega_1$, $\theta_{L|L} = 1 - \omega_1$, $\omega_1 \in (0,1)$ is an equilibrium announcement strategy, with associated trading strategy $\mathcal{T}_{(-,H,H)} = 1, \mathcal{T}_{(-,H,L)} = 1, \mathcal{T}_{(-,L,L)} = -\delta$.

The more P_0 decreases, the more ω_1 has to increase. As $\omega_1 \uparrow 1$, the announcement becomes uninformative, since the manipulator always announces $a = H$, and so it is effectively the same as not announcing at all. If P_0 decreases enough, the above condition no longer holds and the manipulator switches to not announcing (or announcing a completely uninformative announcement). It then follows that $\bar{P}^1 \equiv \frac{\bar{P}^N_{(-,-,L)}+\delta\bar{P}^T_{(-,L,L)}}{1+\delta}$. Obviously, as $P_0 \uparrow \bar{P}^2$ we have $\omega_1 \downarrow 0$.

Downward manipulation (DM): $P_0 \in \left(\bar{P}^3, \bar{P}^4\right)$

This case is identical to the previous one and is omitted.

Never announces (NA): $P_0 \in \left(-\infty, \bar{P}^1\right] \cup \left[\bar{P}^4, +\infty\right)$

If $\theta_{N|H} = \theta_{N|L} = 1$, then from parts (ii) and (iii) of lemma 1 we have

$$\bar{P}_{(-,H,s_M)} = \bar{P}_{(-,N,s_M)} = \bar{P}_{(-,L,s_M)},\ s_M \in \{H, L\}, \tag{6}$$

since $\frac{\theta_{H|H}}{\theta_{H|L}} = \frac{\theta_{N|H}}{\theta_{N|L}} = \frac{\theta_{L|H}}{\theta_{L|L}} = 1$ (trembles are equally likely to occur when $s_M = H$ or $s_M = L$).

This implies that the manipulator does not deviate from $\theta_{N|H} = \theta_{N|L} = 1$ which is the equilibrium announcement strategy. Notice that it does not depend on P_0, which implies that never announcing is always an equilibrium. P_0only determines the trading strategy part. We have $\mathcal{T}_{(-,N,H)} = 1$ if $P_0 \le \bar{P}^N_{(-,-,H)} \le \bar{P}^4$ and $\mathcal{T}_{(-,N,H)} = -\delta$ otherwise; and $\mathcal{T}_{(-,N,L)} = 1$ if $P_0 \le \bar{P}^N_{(-,-,L)} \le \bar{P}^1$ and $\mathcal{T}_{(-,N,L)} = -\delta$ otherwise. Therefore, $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,H,L)} = 1$ if $P_0 \in \left(-\infty, \bar{P}^1\right]$ and $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,H,L)} = -\delta$ if $P_0 \in \left[\bar{P}^4, +\infty\right)$.

Other equilibria

What matters in the manipulator's announcement strategy, is not what he announces ($a = H$, $a = L$ or $a = N$) when $s_M = H$ or $s_M = L$, but the relative frequency with which he announces something when $s_M = H$ vs. when $s_M = L$. Therefore, all the above equilibria have equivalent equilibria where the manipulator effectively provides the same information but through different announcements strategies. As an example, take the truthful announcement, $\theta_{H|H} = \theta_{L|L} = 1$. Equivalent equilibria are $\theta_{a|L} = \theta_{a'|H} = 1, \forall a \neq a'$, of which $\theta_{L|H} = \theta_{N|L} = 1$ is one example. In this case, $a = L$ signals $s_M = H$ and $a = N$ signals $s_M = L$. Because there is no penalty for lying, signaling $s_M = H$ with $a = L$ is the same as signaling it with $a = H$ or $a = N$ (both are penalty-free). But it doesn't end here. a and a' can be mixed strategies. One example is $\theta_{H|H} = 1, \theta_{L|L} = 1/2, \theta_{N|L} = 1/2$ in which case both $a = L$ and $a = N$ signal for $s_M = L$. Since these equilibria are equivalent, I choose to focus on the one where the manipulator uses $a = H$ to signal $s_M = H$ and $a = L$ to signal $s_M = L$.

Finally, recall that never announcing (or the equivalent uninformative announcement) is always an equilibrium. However, any equilibrium where the announcement is (at least) partially informative is preferred by the manipulator, since he is able to influence prices in a favorable way. Therefore, when there is no penalty never announcing is the focal equilibrium only when it is the only equilibrium. ■

Proof of theorem 2. The proof is in everything similar to the one of theorem 1. The only two differences introduced by the penalty k are: the manipulator may prefer to never announce instead of manipulating excessively or even announcing truthfully, even if the latter are equilibria; and if $a = N$ is a zero-probability event, at some point it is optimal to deviate from lying to $a = N$, which causes a jump in the price associated to $a = N$. To avoid this problem, I focus on strategies where $a = L$ ($a = H$) and $a = N$ are substitutes for signaling $s_M = L$ ($s_M = H$) when P_0 is below (above) some threshold. Equilibria where $a = N$ is never played, if they exist, are equivalent. Without loss of generality, I focus on the case where $a = N$ is played with almost zero probability $\varepsilon \gtrapprox 0$. I start by identifying the different type of equilibria and the values of P_0 for which they exist. Then I identify the subset of P_0 values for which each equilibrium is preferred, i.e., $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$. Until there, $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$ do not take into account that the never announce equilibrium may be preferred to the other equilibria.

Truthful announcement (TA1): $P_0 \in \left[\bar{P}^2, \bar{P}^{2.5}\right]$

If $\theta_{H|H} = 1$, $\theta_{L|L} = 1 - \varepsilon$, $\theta_{N|L} = \varepsilon$, then from parts (ii) and (iii) of lemma 1 we have

$$\bar{P}_{(-,H,s_M)} > \bar{P}_{(-,N,s_M)} = \bar{P}_{(-,L,s_M)},\ s_M \in \{H, L\}\,. \tag{7}$$

It is easy to verify that $\mathcal{T}_{(-,H,H)} = -\delta$ is inconsistent with $\theta_{H|H} = 1$ in equilibrium. Therefore, it must be that $\mathcal{T}_{(-,H,H)} = 1$, which implies that $\bar{P}_{(-,H,H)} > P_0$. If the manipulator deviates from $\theta_{H|H} = 1$ he does so to $a = N$ in order to avoid the penalty. It is straightforward to obtain that $\Pi_{(-,H,H)} \geq \Pi_{(-,N,H)}$, and so the manipulator does not deviate, whenever $\bar{P}_{(-,H,H)} - P_0 \geq \delta\left(P_0 - \bar{P}_{(-,N,H)}\right) = \delta\left(P_0 - \bar{P}_{(-,L,H)}\right)$.

Now, assume that $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = -\delta$ which implies that $\bar{P}_{(-,L,L)} < P_0$. In this case $\Pi_{(-,L,L)} = \Pi_{(-,N,L)} \geq \Pi_{(-,H,L)}$, and so the manipulator does not deviate from $\theta_{L|L} = 1 - \varepsilon$ and $\theta_{N|L} = \varepsilon$, if $\delta\left(P_0 - \bar{P}_{(-,L,L)}\right) \geq \bar{P}_{(-,H,L)} - P_0 - k$. Finally, assume that $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = 1$, implying that $\bar{P}_{(-,L,L)} > P_0$. Then, $\Pi_{(-,L,L)} = \Pi_{(-,N,L)} \geq \Pi_{(-,H,L)}$ if $\bar{P}_{(-,L,L)} - P_0 \geq \bar{P}_{(-,H,L)} - P_0 - k$.

Bringing all conditions together we have that

$$\bar{P}^2 \leq P_0 \leq \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^T_{(-,L,H)}}{1+\delta},\ \bar{P}^2 = \begin{cases} \frac{\bar{P}^T_{(-,H,L)} + \delta\bar{P}^T_{(-,L,L)} - k}{1+\delta} & if\, k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ -\infty & otherwise \end{cases}$$

and

$$\mathcal{T}_{(-,H,H)} = 1,\ \mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = \begin{cases} -\delta & if\, k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ 1 & otherwise \end{cases}.$$

Truthful announcement (TA2): $P_0 \in \left[\bar{P}^{2.5}, \bar{P}^3\right]$

Following the same steps as above, we obtain

$$\frac{\bar{P}^T_{(-,H,L)} + \delta\bar{P}^T_{(-,L,L)}}{1+\delta} \leq P_0 \leq \bar{P}^3,\ \bar{P}^3 = \begin{cases} \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^T_{(-,L,H)} + \delta k}{1+\delta} & if\, k < \bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} \\ +\infty & otherwise \end{cases}$$

and

$$\mathcal{T}_{(-,L,L)} = 1,\ \mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,N,H)} = \begin{cases} 1 & if\, k < \bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} \\ -\delta & otherwise \end{cases}$$

It then follows that $\bar{P}^{2.5}$ is any value satisfying

$$\frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)}}{1+\delta} \leq \bar{P}^{2.5} \leq \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta}.$$

Upward manipulation (UM): $P_0 \in \left(\bar{P}^1, \bar{P}^2\right)$

Suppose that $P_0 < \bar{P}^2$. Following the same steps as in the proof of theorem 1, we obtain that $\theta_{H|H} = 1$, $\theta_{H|L} = \omega_1$, $\theta_{L|L} = 1-\omega_1-\varepsilon$, $\theta_{N|L} = \varepsilon$, $\omega_1 \in (0,1)$ is an equilibrium announcement strategy, since

$$\frac{\bar{P}_{(-,H,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta} = P_0 \leq \frac{\bar{P}_{(-,H,H)} + \delta \bar{P}^T_{(-,L,H)}}{1+\delta}.$$

If $P_0 > \bar{P}^T_{(-,L,L)}$ the associated trading strategy is $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = 1$, $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = -\delta$, otherwise it is $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = \mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = 1$.

As long as $P_0 > \bar{P}^T_{(-,L,L)}$, if P_0 decreases, ω_1 has to increase in order for $\bar{P}_{(-,H,L)}$ to decrease and the condition above to hold (obviously, as $P_0 \uparrow \bar{P}^2$ we have $\omega_1 \downarrow 0$.). When $\omega_1 = 1$, $\bar{P}_{(-,H,L)} = \bar{P}^N_{(-,-,L)}$ and so can't decrease any further. Therefore, we must have $P_0 \geq \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta} > \bar{P}^T_{(-,L,L)}$ for the condition above to hold and an upward manipulation equilibrium to exist. The second inequality holds whenever $k < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$.

If, however, $P_0 = \bar{P}^T_{(-,L,L)} \geq \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta}$, the equilibrium strategy for any $P_0 < \bar{P}^T_{(-,L,L)}$ is the same as that for $P_0 = \bar{P}^T_{(-,L,L)}$. To see this, notice that, since the trading strategy is to always take a long position, the manipulator is indifferent between $a = H$ and $a = L$ when $s_M = L$ (hence $\omega_1 \in (0,1)$) if $\bar{P}_{(-,H,L)} - P_0 - k = \bar{P}^T_{(-,L,L)} - P_0 \Leftrightarrow \bar{P}_{(-,H,L)} = \bar{P}^T_{(-,L,L)} + k$, which does not depend on P_0.

Notice that if $k \geq \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)}$, there is no upward manipulation equilibrium, since we are back in the case where the manipulator announces truthfully ($\bar{P}^2 = -\infty$). It then follows that

$$\bar{P}^1 \quad = \quad \begin{cases} \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,L,L)} - k}{1+\delta} < \bar{P}^2 & if\, k < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \\ -\infty < \bar{P}^2 & if\, \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \leq k < \bar{P}^T_{(-,H,L)} - \bar{P}^T_{(-,L,L)} \\ -\infty = \bar{P}^2 & otherwise \end{cases}.$$

Downward manipulation (DM): $P_0 \in \left(\bar{P}^3, \bar{P}^4\right)$

Proceeding in the same way as above, we obtain

$$\bar{P}^4 \quad = \quad \begin{cases} \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)} + \delta k}{1+\delta} > \bar{P}^3 & if\, k < \bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} \\ +\infty > \bar{P}^3 & if\, \bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} \leq k < \bar{P}^T_{(-,H,H)} - \bar{P}^T_{(-,L,H)} \\ +\infty = \bar{P}^3 & otherwise \end{cases}.$$

Never announces (NA): $P_0 \in \left(-\infty, \bar{P}^1\right] \cup \left[\bar{P}^4, +\infty\right)$

Like in the case of $k = 0$, never announcing is always an equilibrium.

Preferred equilibria: determination of $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$

Unlike the case of $k = 0$, there are cases where the never announce equilibrium is preferred to the other equilibria. To determine the threshold prices $\bar{P}^1$, $\bar{P}^2$, $\bar{P}^3$ and $\bar{P}^4$ I compare the expected utility of never announcing with that of other equilibria.

Starting with $\bar{P}^1$, there are three cases to be considered, corresponding to the combinations of possible trading strategies in both equilibria. The first case is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = 1$, $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the UM equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. The NA equilibrium is preferred if

$$\frac{\bar{P}^N_{(-,-,H)} - P_0 + \delta\left(P_0 - \bar{P}^N_{(-,-,L)}\right)}{2} > \frac{\bar{P}_{(-,H,H)} - P_0 + \omega_1\left(\bar{P}_{(-,H,L)} - P_0 - k\right) + (1-\omega_1)\delta\left(P_0 - \bar{P}^T_{(-,L,L)}\right)}{2}$$
$$\Leftrightarrow \frac{\bar{P}^N_{(-,-,H)} - P_0 + \delta\left(P_0 - \bar{P}^N_{(-,-,L)}\right)}{2} > \frac{\bar{P}_{(-,H,H)} - P_0 + \delta\left(P_0 - \bar{P}^T_{(-,L,L)}\right)}{2}$$
$$\Leftrightarrow \delta\left(\bar{P}^T_{(-,L,L)} - \bar{P}^N_{(-,-,L)}\right) > \bar{P}_{(-,H,H)} - \bar{P}^N_{(-,-,H)}$$

which, from parts (ii) and (iii) of lemma 1 and the definition of the variables, is impossible. The second line follows from the fact that if $\omega_1 \in (0,1)$ then the utility from $a = H$ or $a = L$ when $s_M = L$ is the same.

The second case, is when the trading strategy in the UM equilibrium is as above and $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$ in the NA equilibrium. Comparing expected utilities, the NA equilibrium is preferred if P_0 satisfies

$$\bar{P}^T_{(-,L,L)} \leq P_0 < \frac{\bar{P}^N_{(-,-,L)} + \delta\bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}_{(-,H,H)}}{1+\delta} < \bar{P}^N_{(-,-,L)}$$

provided that

$$\frac{\bar{P}^N_{(-,-,L)} + \delta\bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}_{(-,H,H)}}{1+\delta} > \bar{P}^T_{(-,L,L)} \Leftrightarrow \bar{P}_{(-,H,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)}$$

and so the trading strategy in the UM equilibrium is the assumed one. Notice that $\bar{P}^N_{(-,-,H)}$ is the UM equilibrium value, and so it is implicit that the UM equilibrium exists.

The third and final case, is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,H,L)} = \mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,N,L)} = 1$ in the UM equilibrium which implies that $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$ in the NA equilibrium (since $\bar{P}^T_{(-,L,L)} < \bar{P}^N_{(-,-,L)}$). The NA equilibrium is preferred if

$$\bar{P}_{(-,H,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)}.$$

This means is that if the NA equilibrium is preferred to a UM equilibrium when $P_0 \geq \bar{P}^T_{(-,L,L)}$, it will also be preferred to the UM equilibria $\forall P_0 < \bar{P}^T_{(-,L,L)}$. On the other hand, if the NA equilibrium is never preferred to a UM equilibrium at any $P_0 \geq \bar{P}^T_{(-,L,L)}$, then same is true $\forall P_0 < \bar{P}^T_{(-,L,L)}$. It then follows that

$$\bar{P}^1 = \begin{cases} \frac{\bar{P}^N_{(-,-,L)} + \delta\bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}^*_{(-,H,H)}}{1+\delta} & if\ \bar{P}^1 > \bar{P}^T_{(-,L,L)} \\ -\infty & otherwise \end{cases}$$

where $\bar{P}^*_{(-,H,H)}$ is the equilibrium value in the UM equilibrium when $P_0 = \bar{P}^1$.

Now, looking at $\bar{P}^2$, there are three possible cases. In the first case, $\mathcal{T}_{(-,H,H)} = 1$, $\mathcal{T}_{(-,L,L)} = -\delta$

in the TA1 equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. The NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} < \bar{P}^T_{(-,L,L)} - \bar{P}^N_{(-,-,L)}$$

which, from parts (ii) and (iii) of lemma 1, is impossible.

In the second case, the trading strategy in the NA equilibrium changes to $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$. Then, the NA equilibrium is preferred if

$$\begin{aligned} P_0 &< \frac{\bar{P}^N_{(-,-,H)} + \delta \bar{P}^T_{(-,L,L)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,H,H)}}{1+\delta} \\ &= \bar{P}^T_{(-,L,L)} + \frac{\bar{P}^N_{(-,-,H)} - \bar{P}^T_{(-,H,H)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}}{1+\delta} \\ &\leq \bar{P}^T_{(-,L,L)} \end{aligned}$$

which is impossible. The last inequality follows from part (v) of lemma 1 and is due to the risk aversion and reduction of uncertainty associated to the TA1 equilibrium.

In the third case, $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = 1$ in the TA1 equilibrium which implies that $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = 1$ in the NA equilibrium. In this case the NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$$

which once again is impossible due to part (v) of lemma 1. Therefore, the NA equilibrium is never preferred to the TA1 equilibrium and $\bar{P}^2$ is as defined previously.

Turning to $\bar{P}^3$there are three possible cases. One of the cases is when $\mathcal{T}_{(-,H,H)} = 1$, $\mathcal{T}_{(-,L,L)} = -\delta$ in the TA2 (or TA1) equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. We have just seen that in this case the NA equilibrium is never preferred to the TA2 equilibrium. The second case is when $\mathcal{T}_{(-,H,H)} = 1$, $\mathcal{T}_{(-,L,L)} = -\delta$ in the TA2 equilibrium and $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. In this case the NA equilibrium is preferred if P_0 satisfies

$$P_0 > \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^T_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)}}{1+\delta} > \bar{P}^N_{(-,-,H)}. \tag{8}$$

Due to part (v) of lemma 1 we have

$$\begin{aligned} &\frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^T_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)}}{1+\delta} \\ = \; &\bar{P}^T_{(-,H,H)} + \delta \frac{\bar{P}^N_{(-,-,H)} - \bar{P}^T_{(-,H,H)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}}{1+\delta} \\ < \; &\bar{P}^T_{(-,H,H)} \end{aligned}$$

and so the assumed trading strategies are optimal.

The third case is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = -\delta$ in the TA2 equilibrium which implies that $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. The NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} > \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$$

which holds true due to part (v) of lemma 1.

The $\bar{P}^3$ is then given by the threshold (8) or by the threshold of existence of the TA2 equilibrium, whichever is smaller,

$$\bar{P}^3 = \begin{cases} \frac{\bar{P}^T_{(-,H,H)}+\delta\bar{P}^T_{(-,L,H)}+\delta k}{1+\delta} & if\, k < \bar{P}^N_{(-,-,H)} - \bar{P}^T_{(-,L,H)} + \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)} \\ \frac{\bar{P}^T_{(-,H,H)}+\delta\bar{P}^N_{(-,-,H)}-\delta\bar{P}^T_{(-,L,L)}+\delta\bar{P}^N_{(-,-,L)}}{1+\delta} & otherwise \end{cases}.$$

Finally, to determine $\bar{P}^4$ we need to look at three cases. In the first, we have $\mathcal{T}_{(-,L,L)} = \mathcal{T}_{(-,L,H)} = -\delta$, $\mathcal{T}_{(-,H,H)} = 1$ in the DM equilibrium and $\mathcal{T}_{(-,N,H)} = 1$, $\mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. The NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} < \bar{P}^N_{(-,-,L)} - \bar{P}^T_{(-,L,L)}$$

which is impossible due to part (v) of lemma 1.

In the second case, the trading strategy in the NA equilibrium changes to $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = -\delta$. In this case the NA equilibrium is preferred if P_0 satisfies

$$\bar{P}^T_{(-,H,H)} \geq P_0 > \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^N_{(-,-,H)} - \delta\bar{P}_{(-,L,L)} + \delta\bar{P}^N_{(-,-,L)}}{1+\delta} > \bar{P}^N_{(-,-,H)}.$$

Due to part (v) of lemma 1 the threshold above is below $\bar{P}^T_{(-,H,H)}$ and so there are P_0values that satisfy the condition while the assumed strategies are optimal. Notice that $\bar{P}_{(-,L,L)}$is the DM equilibrium value and so it is implicit that it exists.

Finally, the third case is when $\mathcal{T}_{(-,H,H)} = \mathcal{T}_{(-,L,L)} = -\delta$ in the DM equilibrium which implies that $\mathcal{T}_{(-,N,H)} = \mathcal{T}_{(-,N,L)} = -\delta$ in the NA equilibrium. In that case, the NA equilibrium is preferred if

$$\bar{P}^T_{(-,H,H)} - \bar{P}^N_{(-,-,H)} > \bar{P}^N_{(-,-,L)} - \bar{P}_{(-,L,L)}$$

which always holds true due to parts (ii), (iii) and (v) of lemma 1. It then follows that

$$\bar{P}^4 = \frac{\bar{P}^T_{(-,H,H)} + \delta\bar{P}^N_{(-,-,H)} - \delta\bar{P}^*_{(-,L,L)} + \delta\bar{P}^N_{(-,-,L)}}{1+\delta}$$

where $\bar{P}^*_{(-,L,L)}$ is the equilibrium value in the DM equilibrium when $P_0 = \bar{P}^4$.

This concludes the proof of theorem 2. ∎

Proof of theorem 3. Consider any UM equilibrium and ω_1. The expected utility in this equilibrium is given by

$$\frac{\Pi_{(-,H,H)} + \omega_1\Pi_{(-,H,L)} + (1-\omega_1)\,\Pi_{(-,L,L)}}{2} = \frac{\bar{P}_{(-,H,H)} - P_0 + \omega_1\left(\bar{P}_{(-,H,L)} - P_0 - k\right) + (1-\omega_1)\,\Pi_{(-,L,L)}}{2}.$$

If k increases, then $\Pi_{(-,H,L)}$ decreases. As a result, $\Pi_{(-,H,L)} < \Pi_{(-,L,L)}$ and $\theta_{H|L} = \omega_1$ is no longer an equilibrium. From lemma 1, as ω_1 decreases, $\bar{P}_{(-,H,L)}$ and $\bar{P}_{(-,H,H)}$ increase while $\bar{P}_{(-,L,L)}$ remains constant. Consequently, $\Pi_{(-,H,L)}$ and $\Pi_{(-,H,H)}$ increase, and $\Pi_{(-,L,L)}$ remains constant. An equilibrium is reached when either $\omega_1 \in (0,1)$ is such that $\Pi_{(-,H,L)} = \Pi_{(-,L,L)}$ (a UM equilibrium with less manipulation) or $\omega_1 = 0$ and $\Pi_{(-,H,L)} < \Pi_{(-,L,L)}$ (a TA equilibrium). In both cases the increase in k increases the manipulator's expected utility conditional on $s_M = H$ and has no impact on the expected utility conditional on $s_M = L$. Therefore, unconditionally the manipulator is better off with the increase in k.

The case of a DM equilibrium is very similar. An increase in k leads to a decrease in ω_2. In the new equilibrium the manipulator's expected utility conditional on $s_M = L$ increases while the expected utility conditional on $s_M = H$ remains constant, and so the manipulator is better off with the increase in k.

The cases of a TA and NA equilibria are trivial. In both cases the manipulator pays no penalty and so the increase in k has no impact on these equilibria. Therefore, if the TA (NA) equilibrium was preferred to a NA (TA) equilibrium, an increase k will not change that. If the NA equilibrium was preferred to a UM or DM equilibrium, the increase in the expected utility in UM and DM equilibria may make one of these preferable over the NA equilibrium, which increase the expected utility. If by the contrary, it the NA equilibrium was not preferred to a UM or DM equilibrium, an increase in k will not change that.

Therefore, an increase in k may increase or not impact the manipulator's expected utility, depending on the value of P_0 and the initial level of k, but never decrease it. If k is small enough so that there exist P_0 that support a UM or DM equilibrium, the expected utility averaging over P_0 increases in k. Otherwise, it remains constant. This concludes the proof. ■

Proof of theorem 4. The last part of the theorem is as obvious consequence of the fact that the manipulator can only avoid the penalty by not announcing. Therefore, as k diverges to infinity the expected profit for any strategy other than not announcing becomes negative, and thus smaller than the expected utility of not announcing. Not announcing is then the optimal strategy.

To prove the first part of the theorem, I first derive the optimal announcement strategies for the case of a small k. Using the same method as in the proof of theorem 2, we obtain that the optimal strategy, when k is sufficiently small, is

$$\begin{cases} \theta_{H|H} = 1,\, \theta_{N|L} = 1 & if\, P_0 \in \left[\bar{P}^2, \bar{P}^{2.5}\right] \; (Truthful\ Announcement) \\ \theta_{H|H} = 1,\, \theta_{N|H} = 1 & if\, P_0 \in \left[\bar{P}^{2.5}, \bar{P}^3\right] \; (Truthful\ Announcement) \\ \theta_{H|H} = 1,\, \theta_{H|L} = \omega_1,\, \theta_{N|L} = 1 - \omega_1 & if\, P_0 \in \left(\bar{P}^1, \bar{P}^2\right) \; (Upward\ Manipulation) \\ \theta_{L|L} = 1,\, \theta_{L|H} = \omega_2,\, \theta_{N|H} = 1 - \omega_2 & if\, P_0 \in \left(\bar{P}^3, \bar{P}^4\right) \; (Downward\ Manipulation) \\ \theta_{N|H} = \theta_{N|L} = 1 & if\, P_0 \in \left(-\infty, \bar{P}^1\right] \cup \left[\bar{P}^4, +\infty\right) \; (Never\ Announce) \end{cases}$$

where

$$\begin{aligned} \bar{P}^1 &= \frac{\bar{P}^N_{(-,-,L)} + \delta \bar{P}^T_{(-,N,L)} + \bar{P}^N_{(-,-,H)} - \bar{P}^*_{(-,H,H)} + k\,(1-\rho_M)}{1+\delta} \\ \bar{P}^2 &= \frac{\bar{P}^T_{(-,H,L)} + \delta \bar{P}^T_{(-,N,L)} - k\rho_M}{1+\delta} \\ \bar{P}^{2.5} &\in \left[\frac{\bar{P}^T_{(-,N,L)} + \delta \bar{P}^T_{(-,L,L)} + \delta k\,(1-\rho_M)}{1+\delta}, \frac{\bar{P}^T_{(-,H,H)} + \delta \bar{P}^T_{(-,N,H)} - k\,(1-\rho_M)}{1+\delta}\right] \\ \bar{P}^3 &= \frac{\bar{P}^T_{(-,N,H)} + \delta \bar{P}^T_{(-,L,H)} + \delta k\rho_M}{1+\delta} \\ \bar{P}^4 &= \frac{\bar{P}^T_{(-,N,H)} + \delta \bar{P}^N_{(-,-,H)} - \delta \bar{P}^*_{(-,L,L)} + \delta \bar{P}^N_{(-,-,L)} - k\,(1-\rho_M)}{1+\delta}, \end{aligned}$$

Notice that because truthful announcements are also punished, the manipulator uses $a = N$ to signal $s_M = H$ ($s_M = L$) when P_0 is small (large). By doing so the manipulator avoids being punished while signaling truthfully. Even though truthful signaling occurs less frequently than manipulated signaling in the UM and DM equilibria, the manipulator prefers to avoid the punishment when signaling truthfully. This happens because if he is punished when while sending the manipulated signal, he can commit to manipulate less, which as we have seen benefits the manipulator. By the contrary, being punished for sending the truthful announcement reduces the incentive to being truthful and results in more manipulation which, although not punished, is bad for the manipulator. Because the manipulator always strictly prefers to signal truthfully by not announcing, I have to make an additional assumption on the trembles: when $P_0 \leq \bar{P}^{2.5}$ the manipulator only trembles at $s_M = L$, and when $P_0 > \bar{P}^{2.5}$ only trembles at $s_M = H$. This assumption is needed to avoid the problem that the manipulator may want to deviate to a zero probability announcement whose payoff changes when he does that.

Looking that the expression for $\bar{P}^2$ and $\bar{P}^3$ it is immediate that the former decreases and the latter increases in k. From the expression for $\bar{P}^1$ it looks like it increases in k. However, $\bar{P}^*_{(-,H,H)}$ increases by more than k and so $\bar{P}^1$ actually decreases in k. To see this point, recall from the proof of theorem 3 that if k increases the manipulator decreases the probability with which he manipulates. As a result, both $\bar{P}^*_{(-,H,L)}$ and $\bar{P}^*_{(-,H,H)}$ increase. $\bar{P}^*_{(-,H,L)}$ increases by $\Delta k \rho_M$ in order to restore indifference with announcing $a = L$ when $s_M = L$. It can be shown numerically that $\bar{P}^*_{(-,H,H)}$ in general increases by more than $\bar{P}^*_{(-,H,L)}$. Even when that is not the case, the difference is small. Since $\Delta k (1 - \rho_M) < \Delta k \rho_M$ it follows that in general $\bar{P}^1$ decreases. Using a similar argument, we have that in general $\bar{P}^4$ increases in k. Therefore, when k is small, and increase in k results in a weak increase in the announcement informativeness for all P_0. The improvement in the manipulator's expected utility follows directly from the proof of theorem 3. This concludes the proof. ■

Proof of theorem 5. The proof involves only a lot of algebra and so is omitted. It is available at the reader's request. ■

Proof of theorem 6. Whenever both truthful and informative announcements are more likely as ρ increases, the manipulator's expected utility increases in ρ unless P_0 is so large that the manipulator always takes a short position. Therefore, the manipulator is, on average, better off the larger ρ is.

Consider the case where, prior to the increase in ρ, P_0 supports a TA equilibrium. Assuming $\delta \leq 1$, it can be shown after lots of algebra that

$$\frac{\partial \left[\bar{P}^T_{(-,H,H)} - \delta \bar{P}^T_{(-,L,L)}\right]}{\partial \rho} \geq 0, \; \frac{\partial \left[\bar{P}^T_{(-,H,H)} + \bar{P}^T_{(-,L,L)}\right]}{\partial \rho} > 0.$$

Therefore, the expected utility in a TA equilibrium increases in ρ unless the manipulator always takes a short position.

In the case where, prior to the increase in ρ, P_0 supports an UM or a DM equilibrium, but after the increase in ρ it supports a TA equilibrium, then the expected utility increases. This is because the expected utility of a TA equilibrium is larger than that of any UM or DM equilibrium. Since the expected utility of a TA equilibrium increases in ρ, the result follows.

Finally, in a NA equilibrium where the manipulator always takes long positions, an increase in ρ decreases the residual uncertainty and pushes prices upward. Therefore, the manipulator's expected utility increases. ■

Proof of theorem 7. The proof involves only a lot of algebra and so is omitted. It is available at the reader's request. ■

Proof of theorem 8. The proof involves only a lot of algebra and so is omitted. It is available at the reader's request. ■

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